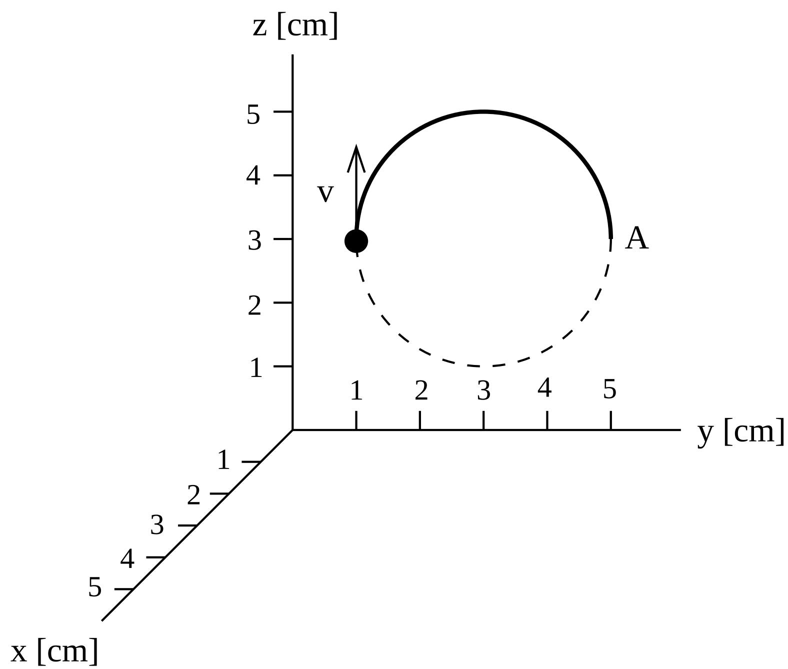
Unit Exam III: Problem #1 (Spring '18)

In a uniform magnetic field of strength $B = 3.5\text{mT}$ [$B = 5.3\text{mT}$], a proton with specifications ($m = 1.67 \times 10^{-27}\text{kg}, \;\; q = 1.60 \times 10^{-19}\text{C}$) moves at speed v around a circle in the yz-plane as shown.

(a) Show that the direction of the magnetic field must be $+\hat{\mathbf{i}}$

(b) What is the speed of the proton?

(c) How long does it take the proton to reach point A from its current position?

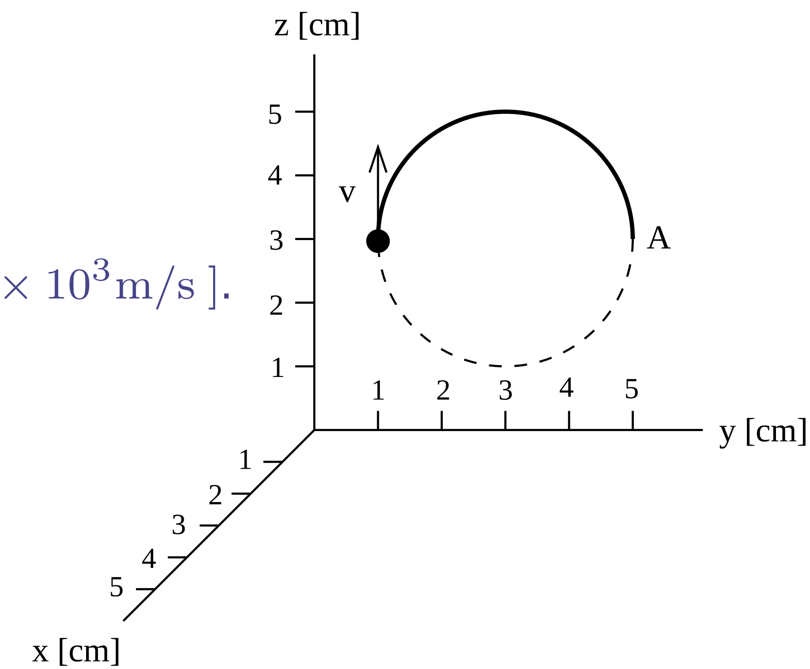


In a uniform magnetic field of strength $B = 3.5\mathrm{mT}$ [$B = 5.3\mathrm{mT}$], a proton with specifications ($m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C}$) moves at speed v around a circle in the yz-plane as shown.

(a) Show that the direction of the magnetic field must be $+\hat{\mathbf{i}}$

(b) What is the speed of the proton?

(c) How long does it take the proton to reach point A from its current position?



Solution:

(a) $F\hat{\mathbf{j}} = qv\hat{\mathbf{k}} \times B\hat{\mathbf{i}}$.

(b) $\dfrac{mv^2}{r} = qvB \quad \Rightarrow\ v = \dfrac{qBr}{m} = 6.71 \times 10^3\mathrm{m/s} \quad [\,10.2 \times 10^3\mathrm{m/s}\,]$.

(c) $t = \dfrac{\pi r}{v} = \dfrac{\pi m}{qB} = 9.37 \times 10^{-6}\mathrm{s} \quad [\,6.19 \times 10^{-6}\mathrm{s}\,]$.

Unit Exam III: Problem #2a (Spring '18)

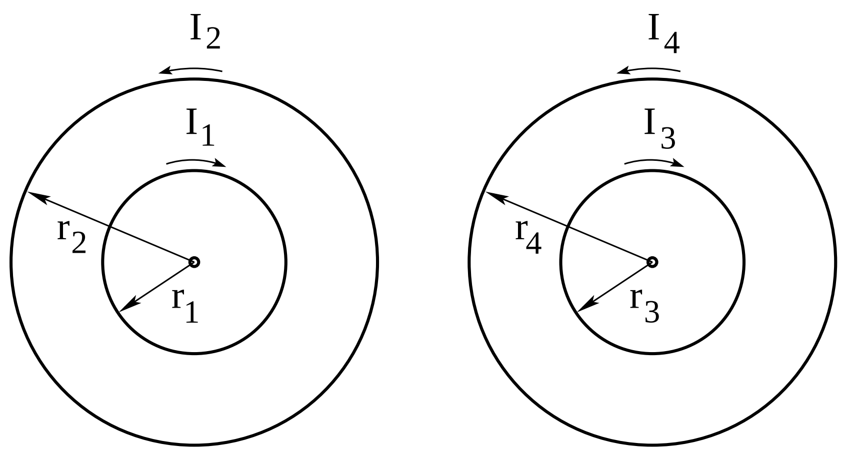
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_1 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_1 = 1.5$A at the center.

(b) Find magnitude μ_4 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_4 = 4.5$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2a (Spring ’18)

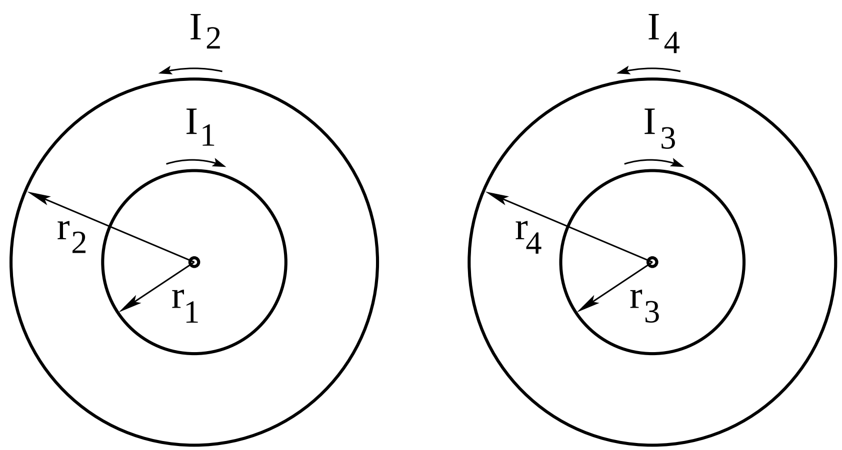
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_1 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_1 = 1.5$A at the center.

(b) Find magnitude μ_4 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_4 = 4.5$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Solution:

(a) $B_1 = \dfrac{\mu_0(1.5\text{A})}{2(5\text{cm})} = 1.88 \times 10^{-5}\text{T} \quad \otimes$

(b) $\mu_4 = \pi(10\text{cm})^2(4.5\text{A}) = 1.41 \times 10^{-1}\text{Am}^2 \quad \odot$

(c) $B_1 = B_2 \quad \Rightarrow \dfrac{I_2}{I_1} = \dfrac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \quad \Rightarrow \dfrac{I_4}{I_3} = \dfrac{r_3^2}{r_4^2} = 0.25.$

Unit Exam III: Problem #2b (Spring '18)

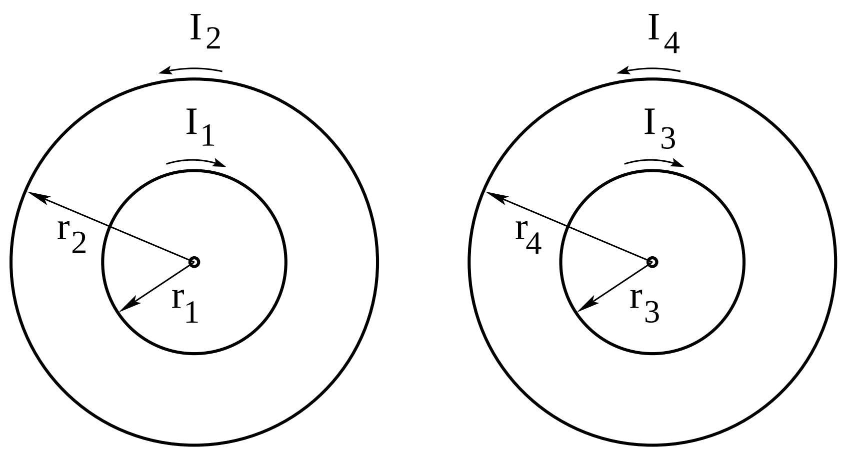
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_2 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_2 = 2.5$A at the center.

(b) Find magnitude μ_3 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_3 = 3$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2b (Spring '18)

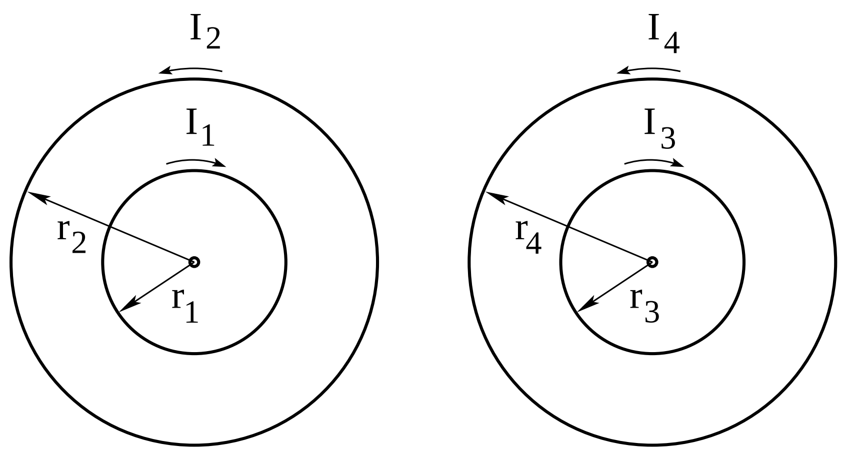
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5\text{cm}$ and $r_2 = r_4 = 10\text{cm}$

(a) Find magnitude B_2 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_2 = 2.5\text{A}$ at the center.

(b) Find magnitude μ_3 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_3 = 3\text{A}$.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Solution:

(a) $B_2 = \dfrac{\mu_0(2.5\text{A})}{2(10\text{cm})} = 1.57 \times 10^{-5}\text{T} \quad \odot$

(b) $\mu_3 = \pi(5\text{cm})^2(3\text{A}) = 2.36 \times 10^{-2}\text{Am}^2 \quad \otimes$

(c) $B_1 = B_2 \quad \Rightarrow \dfrac{I_2}{I_1} = \dfrac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \quad \Rightarrow \dfrac{I_4}{I_3} = \dfrac{r_3^2}{r_4^2} = 0.25.$

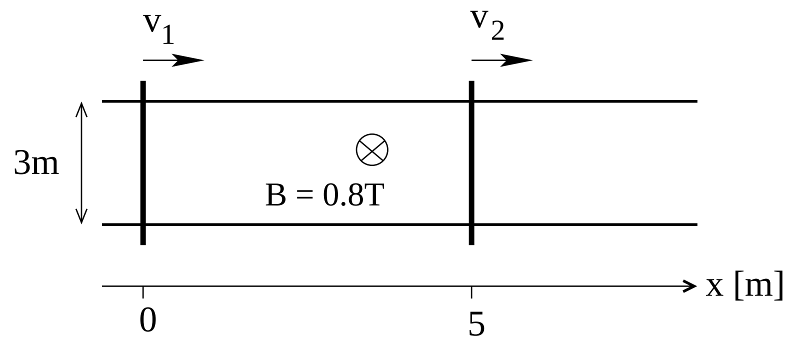
Unit Exam III: Problem #3 (Spring '18)

A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5$m/s, $v_2 = 2.5$m/s [$v_1 = 1.5$m/s, $v_2 = 0.5$m/s].

(a) Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2$s (magnitude only).

(b) Find the induced emf $\mathcal{E}_0$ at time $t = 0$ and $\mathcal{E}_1$ at $t = 2$s (magnitude only).

(c) Find the direction (cw/ccw) of the induced current at $t = 0$.

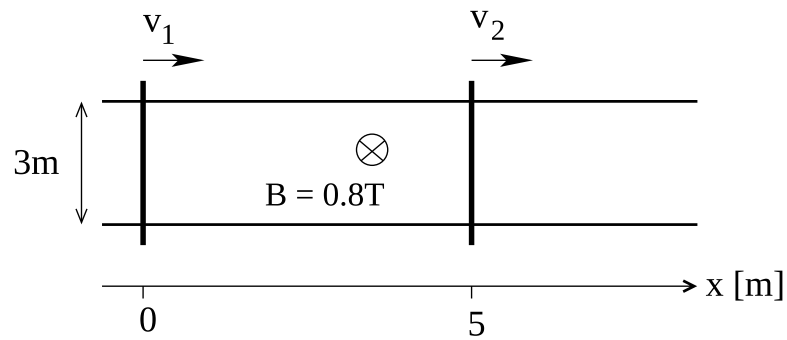


A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5$m/s, $v_2 = 2.5$m/s [$v_1 = 1.5$m/s, $v_2 = 0.5$m/s].

(a) Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2$s (magnitude only).

(b) Find the induced emf $\mathcal{E}_0$ at time $t = 0$ and $\mathcal{E}_1$ at $t = 2$s (magnitude only).

(c) Find the direction (cw/ccw) of the induced current at $t = 0$.



Solution:

(a) $\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}, \quad \Phi_1 = (10\text{m} - 1\text{m})(3\text{m})(0.8\text{T}) = 21.6\text{Wb}$
[$\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}, \quad \Phi_1 = (6\text{m} - 3\text{m})(3\text{m})(0.8\text{T}) = 7.2\text{Wb}$]

(b) $|\mathcal{E}_0| = |\mathcal{E}_1| = (2.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 4.8\text{V}$
[$|\mathcal{E}_0| = |\mathcal{E}_1| = (1.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 2.4\text{V}$]

(c) ccw [cw]