Unit Exam III: Problem #1 (Fall '17)

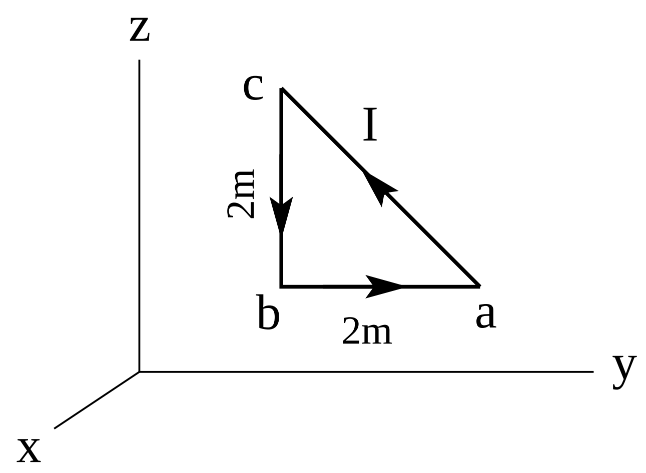
Consider a region with uniform magnetic field $\vec{B} = 4\mathrm{T}\hat{j}$ [$\vec{B} = 5\mathrm{T}\hat{k}$]. A conducting loop in the yz-plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7$A [$I = 0.9$A].

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on the side ab of the loop.

(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on the side bc of the loop.

(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



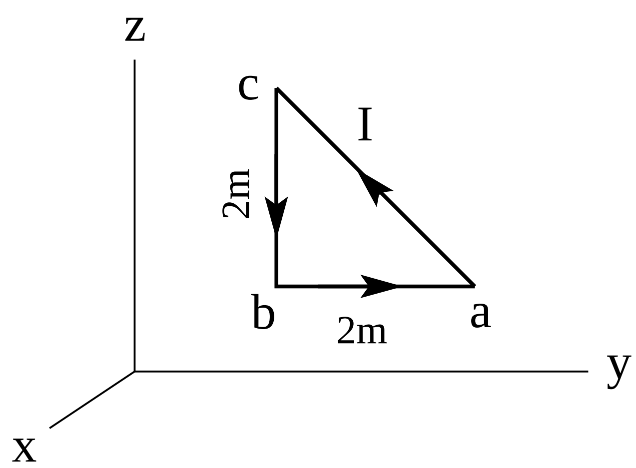
Consider a region with uniform magnetic field $\vec{B} = 4\mathrm{T}\hat{j}$ [$\vec{B} = 5\mathrm{T}\hat{k}$]. A conducting loop in the yz-plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7$A [$I = 0.9$A].

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on the side ab of the loop.

(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on the side bc of the loop.

(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Solution:

(a) $\vec{\mu} = (0.7\mathrm{A})(2\mathrm{m}^2)\hat{i} = 1.4\mathrm{Am}^2\hat{i}$
$[\vec{\mu} = (0.9\mathrm{A})(2\mathrm{m}^2)\hat{i} = 1.8\mathrm{Am}^2\hat{i}]$

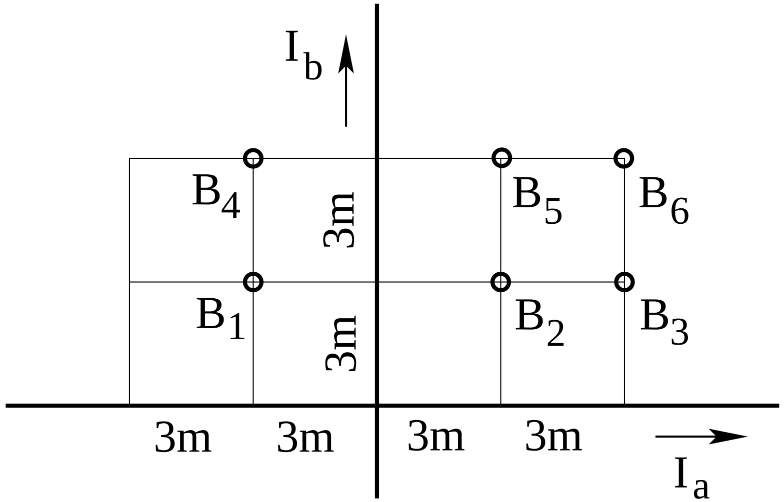
(b) $\vec{F}_{ab} = 0 \quad [\vec{F}_{ab} = (0.9\mathrm{A})(2\mathrm{m}\hat{j}) \times (5\mathrm{T}\hat{k}) = 9.0\mathrm{N}\hat{i}]$

(c) $\vec{F}_{bc} = (0.7\mathrm{A})(-2\mathrm{m}\hat{k}) \times (4\mathrm{T}\hat{j}) = 5.6\mathrm{N}\hat{i} \quad [\vec{F}_{bc} = 0]$

(d) $\vec{\tau} = (1.4\mathrm{Am}^2\hat{i}) \times (4\mathrm{T}\hat{j}) = 5.6\mathrm{Nm}\hat{k}$
$[\vec{\tau} = (1.8\mathrm{Am}^2\hat{i}) \times (5\mathrm{T}\hat{k}) = -9.0\mathrm{Nm}\hat{j}]$

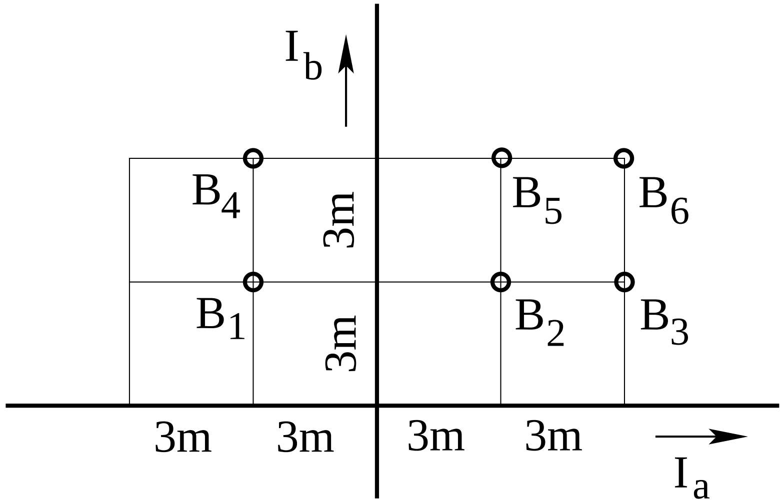
Unit Exam III: Problem #2 (Fall ’17)

Consider two infinitely long, straight wires with currents $I_a = I_b = 7$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, $\mathbf{B}_5$, $\mathbf{B}_6$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall ’17)

Consider two infinitely long, straight wires with currents $I_a = I_b = 7\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3, \mathbf{B}_4, \mathbf{B}_5, \mathbf{B}_6$ at the points marked in the graph.



Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} + \frac{7\text{A}}{3\text{m}}\right) = +0.933\mu\text{T}$ (out of plane).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{3\text{m}}\right) = 0$ (no direction).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{6\text{m}}\right) = +0.233\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} + \frac{7\text{A}}{3\text{m}}\right) = 0.7\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{3\text{m}}\right) = -0.233\mu\text{T}$ (into plane).
- $B_6 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{6\text{m}}\right) = 0$ (no direction).

Unit Exam III: Problem #3 (Fall '17)

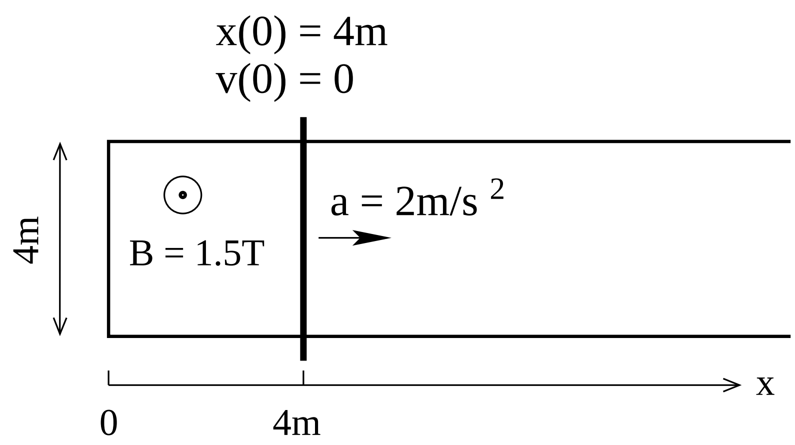
A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

(a) Find the magnetic flux Φ_B through the conducting loop and the induced emf $\mathcal{E}$ around the loop at $t = 0$.

(b) Find the position $x(3\mathrm{s})$ and velocity $v(3\mathrm{s})$ of the rod at time $t = 3$s.

(c) Find the magnetic flux Φ_B through the loop and the induced emf $\mathcal{E}$ around the loop at time $t = 3$s.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Fall ’17)

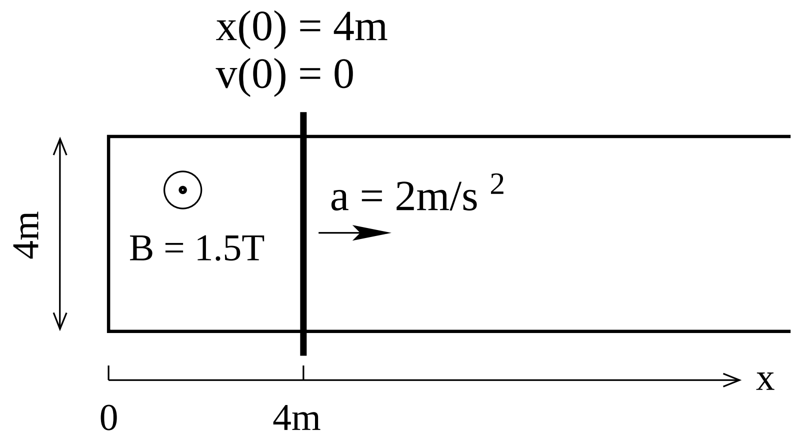
A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

(a) Find the magnetic flux Φ_B through the conducting loop and the induced emf $\mathcal{E}$ around the loop at $t = 0$.

(b) Find the position $x(3\mathrm{s})$ and velocity $v(3\mathrm{s})$ of the rod at time $t = 3$s.

(c) Find the magnetic flux Φ_B through the loop and the induced emf $\mathcal{E}$ around the loop at time $t = 3$s.

Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (16\mathrm{m}^2)(1.5\mathrm{T}) = 24\mathrm{Wb}, \quad \mathcal{E} = 0.$

(b) $x(2\mathrm{s}) = 4\mathrm{m} + \frac{1}{2}(2\mathrm{m/s}^2)(3\mathrm{s})^2 = 13\mathrm{m}, \quad v(3\mathrm{s}) = (2\mathrm{m/s}^2)(3\mathrm{s}) = 6\mathrm{m/s}.$

(b) $\Phi_B = (52\mathrm{m}^2)(1.5\mathrm{T}) = 78\mathrm{Wb}, \quad \mathcal{E} = (6\mathrm{m/s})(1.5\mathrm{T})(4\mathrm{m}) = 36\mathrm{V}.$