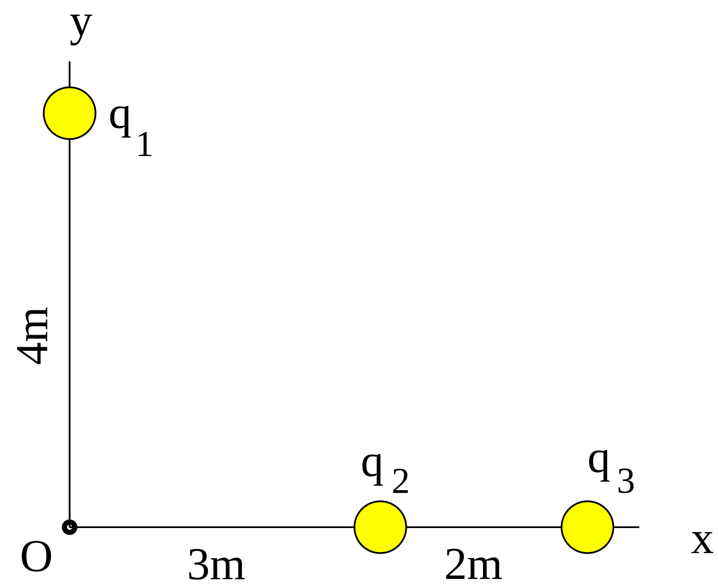
Unit Exam I: Problem #1 (Spring '17)

Point charges $q_1 = +1$nC, $q_2 = +2$nC, $q_3 = -3$nC [$q_1 = -1$nC, $q_2 = +2$nC, $q_3 = +3$nC] are positioned as shown.

(a) Find the components E_x and E_y of the electric field at point O.

(b) Find the electric potential V at point O.

(c) Find the direction $(\uparrow, \nearrow, \rightarrow, \searrow, \downarrow, \swarrow, \leftarrow, \nwarrow)$ of the resultant Coulomb force on charge q_2.



Unit Exam I: Problem #1 (Spring '17)

Point charges $q_1 = +1\text{nC}$, $q_2 = +2\text{nC}$, $q_3 = -3\text{nC}$ [$q_1 = -1\text{nC}$, $q_2 = +2\text{nC}$, $q_3 = +3\text{nC}$] are positioned as shown.
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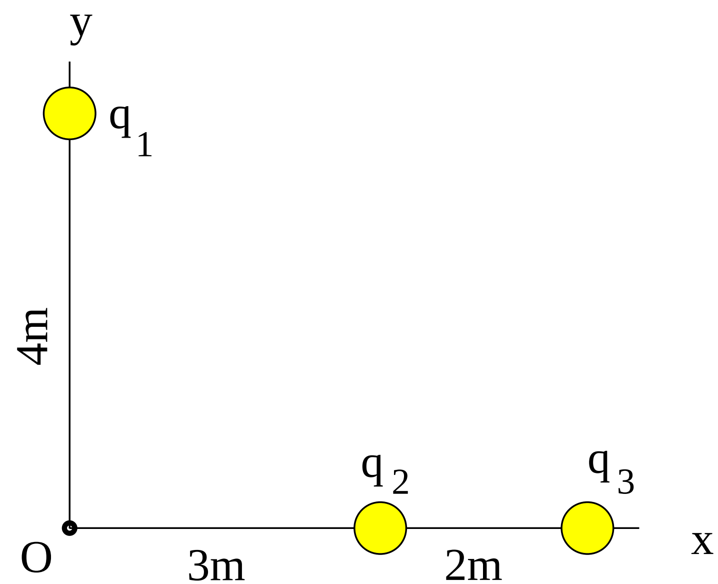


Solution:

(a) $E_x = -k\frac{|q_2|}{(3\text{m})^2} + k\frac{|q_3|}{(5\text{m})^2} = -0.92\,\text{N/C}$

$\left[E_x = -k\frac{|q_2|}{(3\text{m})^2} - k\frac{|q_3|}{(5\text{m})^2} = -3.08\,\text{N/C}\right]$

$E_y = -k\frac{|q_1|}{(4\text{m})^2} = -0.56\,\text{N/C}$

$\left[E_y = +k\frac{|q_1|}{(4\text{m})^2} = +0.56\,\text{N/C}\right]$

(b) $V = k\frac{q_1}{4\text{m}} + k\frac{q_2}{3\text{m}} + k\frac{q_3}{5\text{m}} = 2.85\text{V}$ $\left[V = k\frac{q_1}{4\text{m}} + k\frac{q_2}{3\text{m}} + k\frac{q_3}{5\text{m}} = 9.15\text{V}\right]$

(c) $\searrow$ $[\nwarrow]$

Unit Exam I: Problem #2 (Spring ’17)

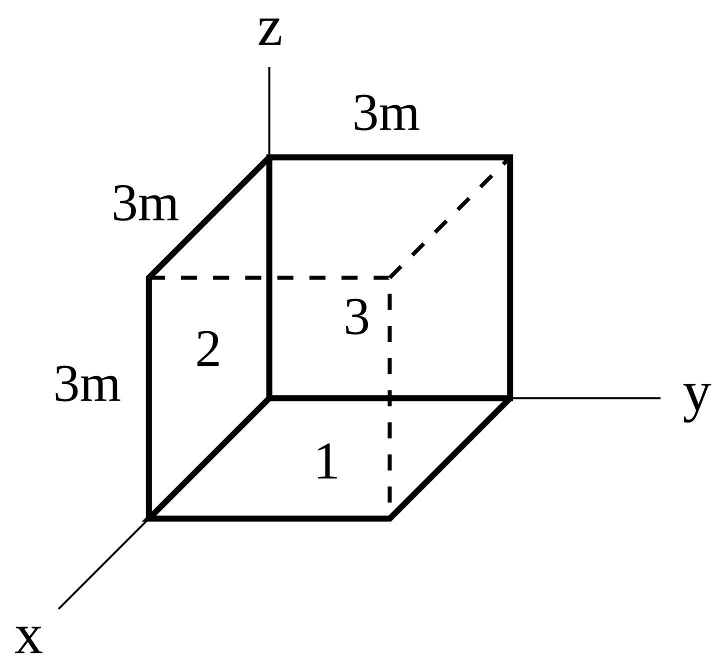
Consider a Gaussian surface in the form of a cube with edges of length 3m placed into a region of uniform electric field $\mathbf{E} = (5\hat{i} - 4\hat{j} + 6\hat{k})$N/C [$\mathbf{E} = (8\hat{i} + 7\hat{j} - 9\hat{k})$N/C].

(a) Find the electric flux $\Phi_E^{(1)}$ through face 1 (in xy plane).

(b) Find the electric flux $\Phi_E^{(2)}$ through face 2 (in xz plane).

(c) Find the electric flux $\Phi_E^{(3)}$ through face 3 (in yz plane).

(d) Find the electric flux $\Phi_E^{(tot)}$ through all six faces added up.



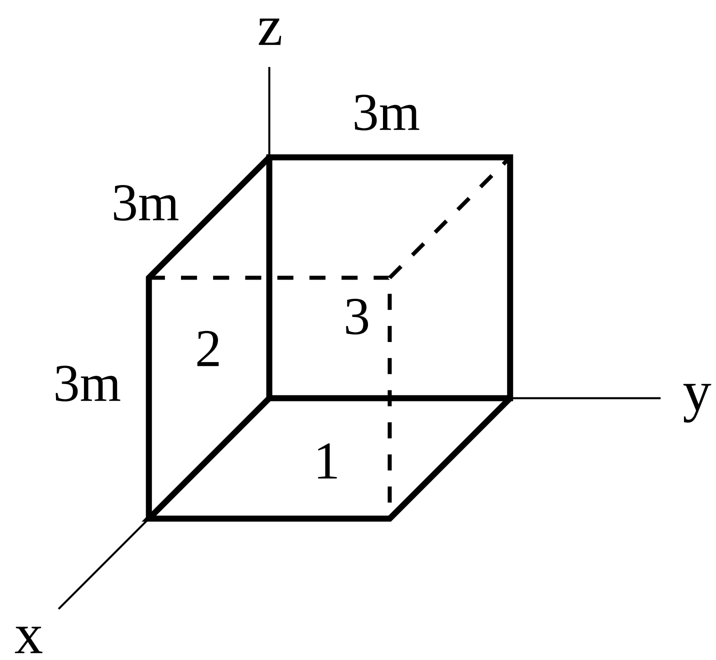
Consider a Gaussian surface in the form of a cube with edges of length 3m placed into a region of uniform electric field $\mathbf{E} = (5\hat{i} - 4\hat{j} + 6\hat{k})$N/C [$\mathbf{E} = (8\hat{i} + 7\hat{j} - 9\hat{k})$N/C].

(a) Find the electric flux $\Phi_E^{(1)}$ through face 1 (in xy plane).

(b) Find the electric flux $\Phi_E^{(2)}$ through face 2 (in xz plane).

(c) Find the electric flux $\Phi_E^{(3)}$ through face 3 (in yz plane).

(d) Find the electric flux $\Phi_E^{(tot)}$ through all six faces added up.



Solution:

(a) $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (6\text{N/C})\hat{k} \cdot (-9\text{m}^2)\hat{k} = -54\text{Nm}^2/\text{C}$

$[\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (-9\text{N/C})\hat{k} \cdot (-9\text{m}^2)\hat{k} = 81\text{Nm}^2/\text{C}]$

(b) $\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-4\text{N/C})\hat{j} \cdot (-9\text{m}^2)\hat{j} = +36\text{Nm}^2/\text{C}$

$[\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (7\text{N/C})\hat{j} \cdot (-9\text{m}^2)\hat{j} = -63\text{Nm}^2/\text{C}]$

(c) $\Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (5\text{N/C})\hat{i} \cdot (-9\text{m}^2)\hat{i} = -45\text{Nm}^2/\text{C}$

$[\Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (8\text{N/C})\hat{i} \cdot (-9\text{m}^2)\hat{i} = -72\text{Nm}^2/\text{C}]$

(d) $\Phi_E^{(tot)} = \dfrac{Q_{in}}{\epsilon_0} = 0 \quad \left[\Phi_E^{(tot)} = \dfrac{Q_{in}}{\epsilon_0} = 0\right]$

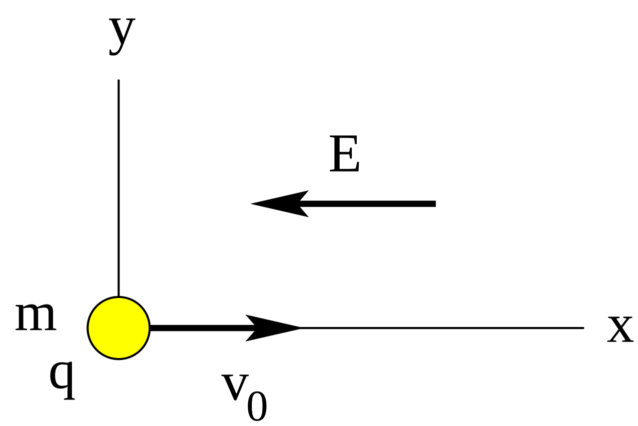
Unit Exam I: Problem #3 (Spring '17)

Consider a region of uniform electric field $\mathbf{E} = -2\mathrm{N/C}\hat{\mathbf{i}}$ [$\mathbf{E} = -3\mathrm{N/C}\hat{\mathbf{i}}$]. A charged particle $(m = 0.04\mathrm{kg},\ q = 6\mathrm{mC})$ [$(m = 0.05\mathrm{kg},\ q = 7\mathrm{mC})$] is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 8\mathrm{m/s}\hat{\mathbf{i}}$ [$\mathbf{v}_0 = 9\mathrm{m/s}\hat{\mathbf{i}}$] from the origin of the coordinate system as shown.

(a) Find the the acceleration a_x of the particle at time $t = 2.5$s.

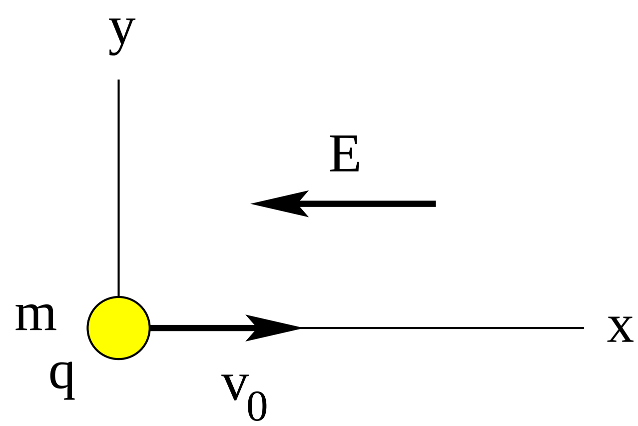
(b) Find its velocity v_x at time $t = 2.5$s.

(c) Find its position x at time $t = 2.5$s.



Consider a region of uniform electric field $\mathbf{E} = -2\mathrm{N/C}\hat{\mathbf{i}}$ [$\mathbf{E} = -3\mathrm{N/C}\hat{\mathbf{i}}$]. A charged particle $(m = 0.04\mathrm{kg},\ q = 6\mathrm{mC})$ [$(m = 0.05\mathrm{kg},\ q = 7\mathrm{mC})$] is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 8\mathrm{m/s}\hat{\mathbf{i}}$ [$\mathbf{v}_0 = 9\mathrm{m/s}\hat{\mathbf{i}}$] from the origin of the coordinate system as shown.

(a) Find the the acceleration a_x of the particle at time $t = 2.5$s.
(b) Find its velocity v_x at time $t = 2.5$s.
(c) Find its position x at time $t = 2.5$s.



Solution:

(a) $a_x = -\frac{q}{m}E = -\frac{6\times 10^{-3}\mathrm{C}}{4\times 10^{-2}\mathrm{kg}}(2\mathrm{N/C}) = -0.3\mathrm{m/s^2}$

$\left[a_x = -\frac{q}{m}E = -\frac{7\times 10^{-3}\mathrm{C}}{5\times 10^{-2}\mathrm{kg}}(3\mathrm{N/C}) = -0.42\mathrm{m/s^2}\right]$

(b) $v_x = v_0 + a_x t = 8\mathrm{m/s} - (0.3\mathrm{m/s^2})(2.5\mathrm{s}) = 7.25\mathrm{m/s}$

$[v_x = v_0 + a_x t = 9\mathrm{m/s} - (0.42\mathrm{m/s^2})(2.5\mathrm{s}) = 7.95\mathrm{m/s}]$

(c) $x = v_0 t + \frac{1}{2}a_x t^2 = (8\mathrm{m/s})(2.5\mathrm{s}) - 0.5(0.3\mathrm{m/s^2})(2.5\mathrm{s})^2 = 19.1\mathrm{m}$

$\left[x = v_0 t + \frac{1}{2}a_x t^2 = (9\mathrm{m/s})(2.5\mathrm{s}) - 0.5(0.42\mathrm{m/s^2})(2.5\mathrm{s})^2 = 21.2\mathrm{m}\right]$