Unit Exam III: Problem #1 (Fall '16)

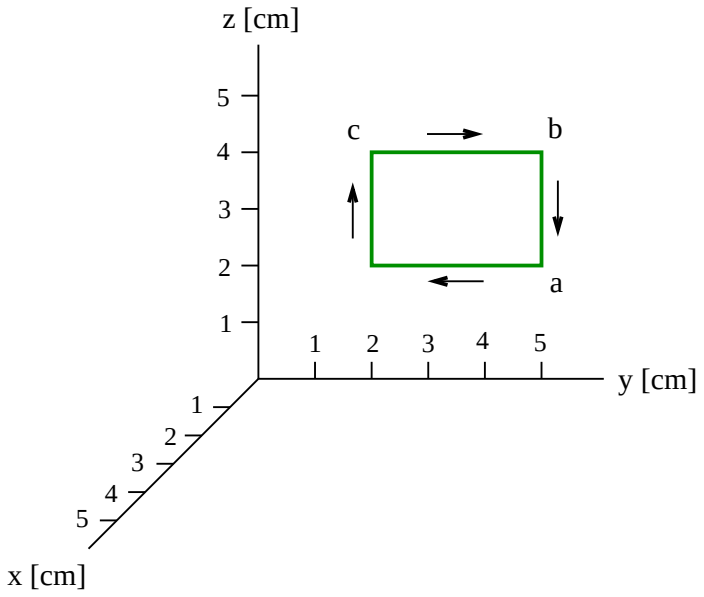
A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.

(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.

(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.

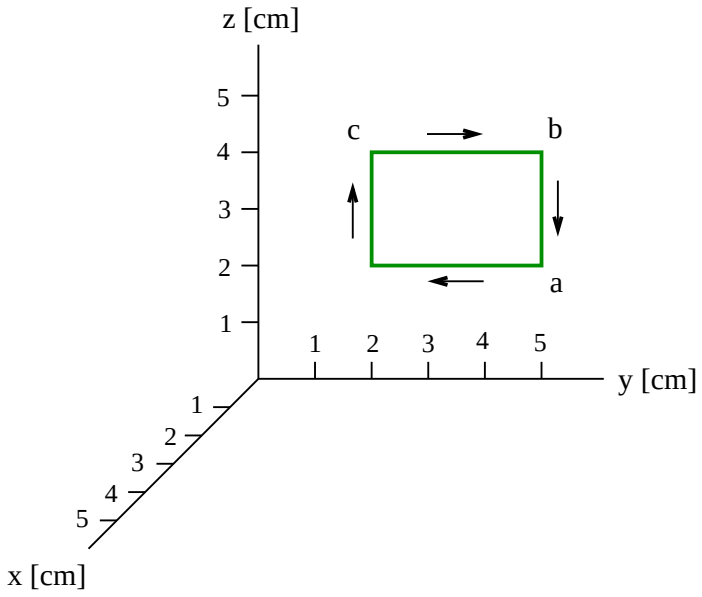
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.

(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



Unit Exam III: Problem #1 (Fall '16)

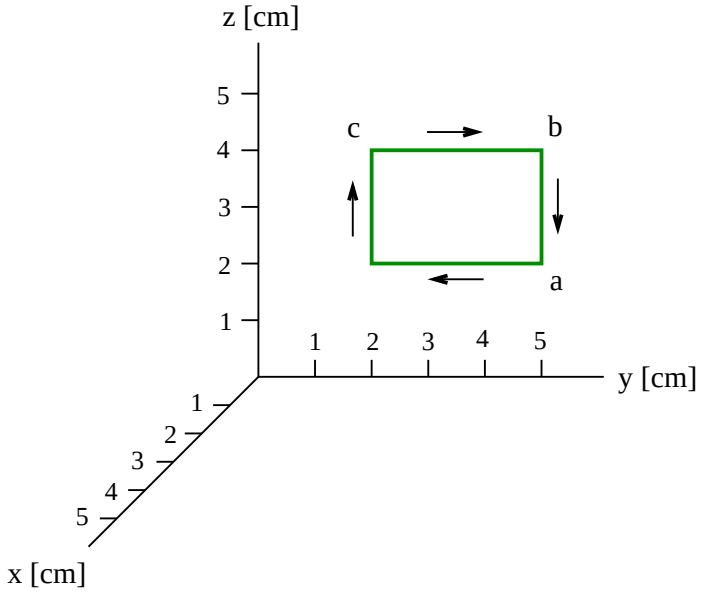
A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



Solution for $I = 1.2\text{A}, \quad \mathbf{B} = 0.7\text{mT}\hat{\mathbf{k}}$:

Unit Exam III: Problem #1 (Fall '16)

A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



(a) $\mathbf{F}_{ab} = (1.2\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 0.$

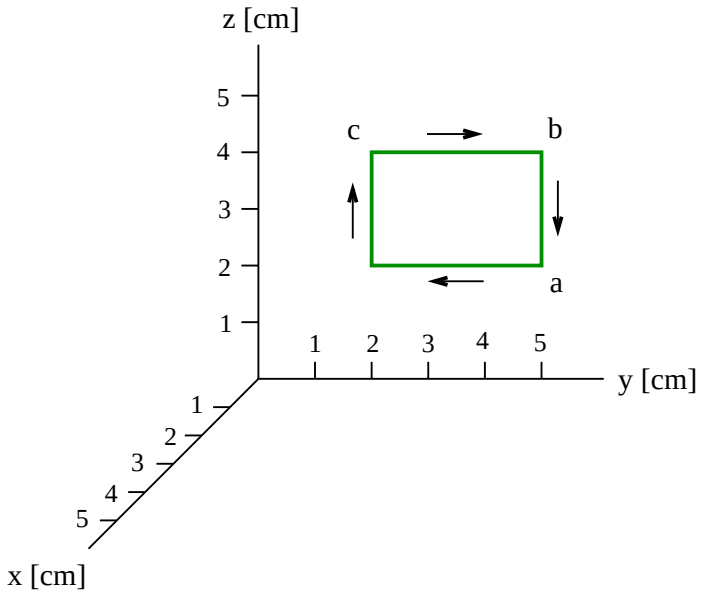
(b) $\mathbf{F}_{bc} = (1.2\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 2.52 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$

(c) $\vec{\mu} = (2\text{cm})(3\text{cm})(1.2\text{A})(-\hat{\mathbf{i}}) = -7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

(d) $\vec{\tau} = (-7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 5.04 \times 10^{-7}\text{Nm}\hat{\mathbf{j}}.$

Unit Exam III: Problem #1 (Fall '16)

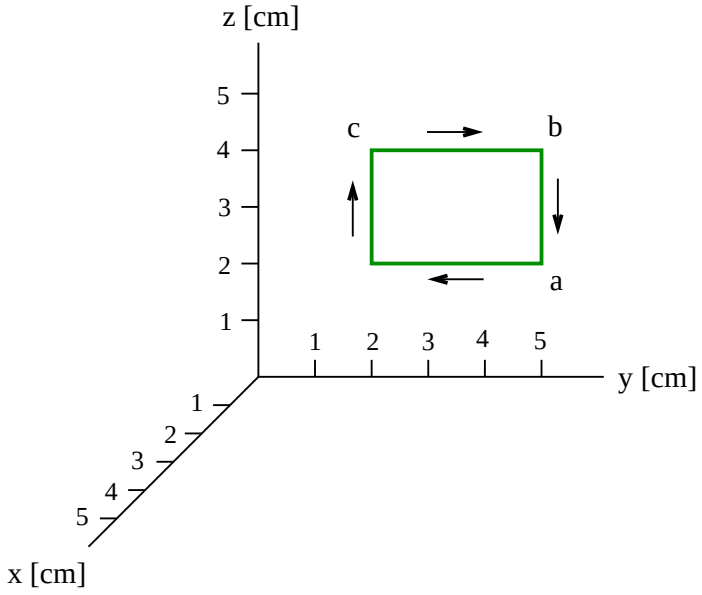
A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



Solution for $I = 2.1\text{A}, \quad \mathbf{B} = 0.8\text{mT}\hat{\mathbf{j}}$

Unit Exam III: Problem #1 (Fall '16)

A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



(a) $\mathbf{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 3.36 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$

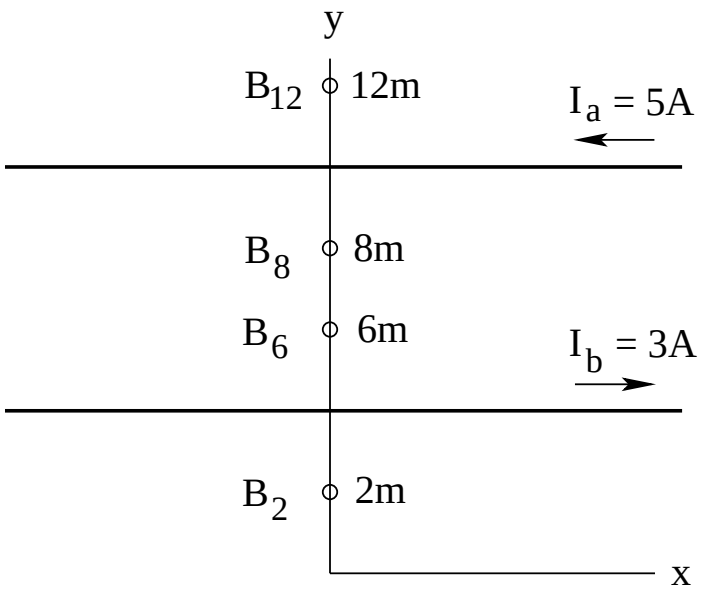
(b) $\mathbf{F}_{bc} = (2.1\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 0.$

(c) $\vec{\mu} = (2\text{cm})(3\text{cm})(2.1\text{A})(-\hat{\mathbf{i}}) = -1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$

(d) $\vec{\tau} = (-1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = -1.01 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.$

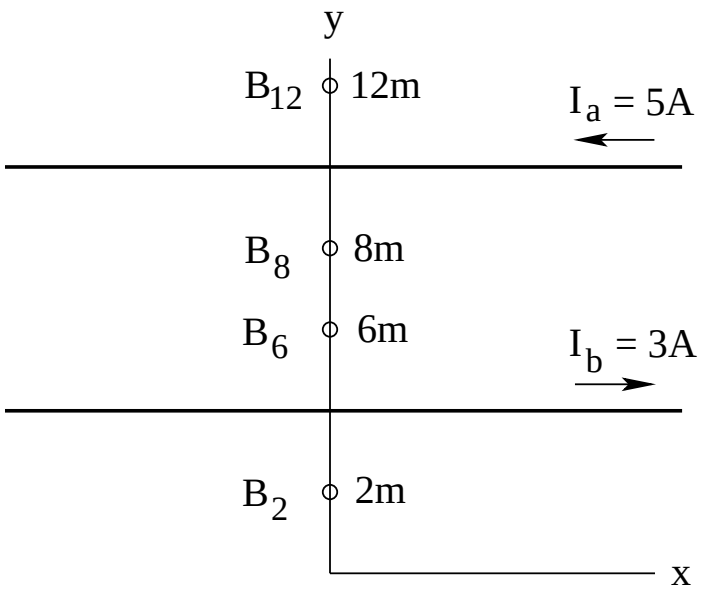
Unit Exam III: Problem #2 (Fall '16)

Two infinitely long, straight wires at positions $y = 10$m and $y = 4$m carry currents I_a and I_b, respectively. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_{12}$, $\mathbf{B}_8$, $\mathbf{B}_6$, and $\mathbf{B}_2$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall '16)

Two infinitely long, straight wires at positions $y = 10\text{m}$ and $y = 4\text{m}$ carry currents I_a and I_b, respectively. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_{12}$, $\mathbf{B}_8$, $\mathbf{B}_6$, and $\mathbf{B}_2$ at the points marked in the graph.

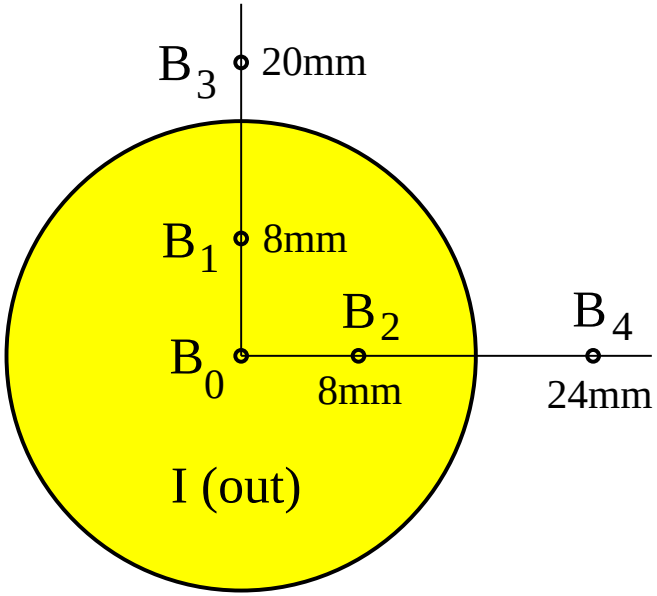


Solution:

- $B_{12} = \dfrac{\mu_0}{2\pi}\left(-\dfrac{5\text{A}}{2\text{m}} + \dfrac{3\text{A}}{8\text{m}}\right) = -4.25 \times 10^{-7}\text{T}$ (in).
- $B_8 = \dfrac{\mu_0}{2\pi}\left(\dfrac{5\text{A}}{2\text{m}} + \dfrac{3\text{A}}{4\text{m}}\right) = 6.50 \times 10^{-7}\text{T}$ (out).
- $B_6 = \dfrac{\mu_0}{2\pi}\left(\dfrac{5\text{A}}{4\text{m}} + \dfrac{3\text{A}}{2\text{m}}\right) = 5.50 \times 10^{-7}\text{T}$ (out).
- $B_2 = \dfrac{\mu_0}{2\pi}\left(\dfrac{5\text{A}}{8\text{m}} - \dfrac{3\text{A}}{2\text{m}}\right) = -1.75 \times 10^{-7}\text{T}$ (in).

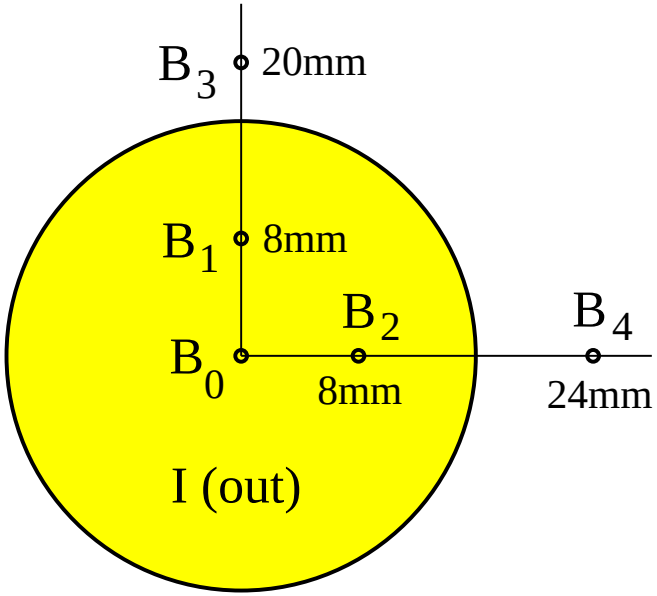
Unit Exam III: Problem #3 (Fall '16)

A conducting wire of 16mm radius carries a current I that is uniformly distributed over its cross section and directed out of the plane. Find direction (left/right/up/down) and magnitude of the magnetic fields $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, and $\mathbf{B}_4$ at the positions indicated if the current is $I = 2.5$A.



Unit Exam III: Problem #3 (Fall '16)

A conducting wire of 16mm radius carries a current I that is uniformly distributed over its cross section and directed out of the plane. Find direction (left/right/up/down) and magnitude of the magnetic fields $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, and $\mathbf{B}_4$ at the positions indicated if the current is $I = 2.5$A.

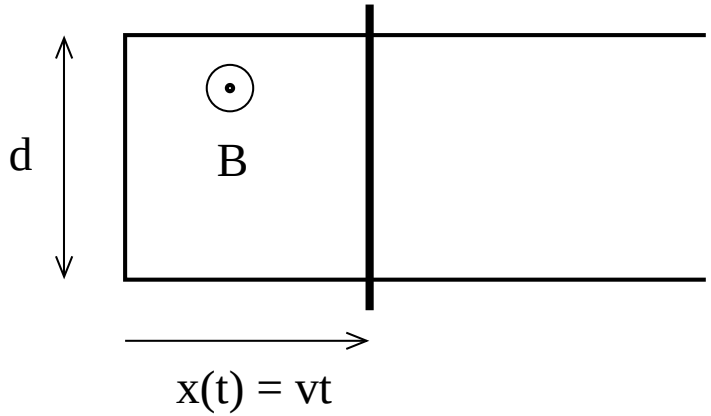


Solution:

- $B_0 = 0$
- $(B_1)(2\pi)(8\text{mm}) = \mu_0(I/4) \quad \Rightarrow \quad B_1 = 1.56 \times 10^{-5}\text{T}$ (left)
- $(B_2)(2\pi)(8\text{mm}) = \mu_0(I/4) \quad \Rightarrow \quad B_2 = 1.56 \times 10^{-5}\text{T}$ (up)
- $(B_3)(2\pi)(20\text{mm}) = \mu_0 I \quad \Rightarrow \quad B_3 = 2.5 \times 10^{-5}\text{T}$ (left)
- $(B_4)(2\pi)(24\text{mm}) = \mu_0 I \quad \Rightarrow \quad B_4 = 2.08 \times 10^{-5}\text{T}$ (up)

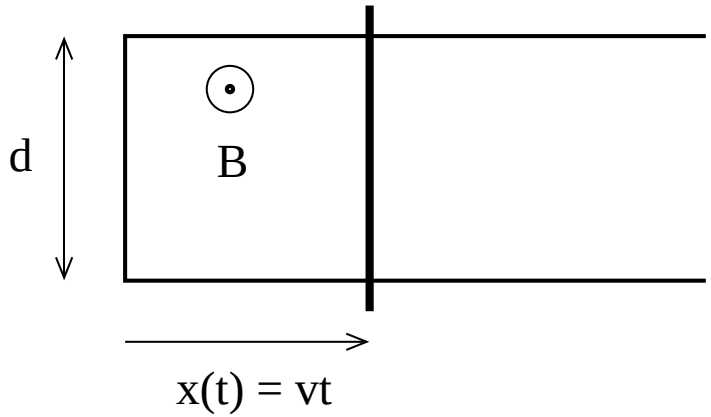
Unit Exam III: Problem #4 (Fall '16)

A conducting frame of width $d = 1.6$m with a moving conducting rod is located in a uniform magnetic field $B = 3$T directed out of the plane. The rod moves at constant velocity $v = 0.4$m/s toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at times $t_2 = 2$s, $t_3 = 3$s, $t_4 = 4$s, and $t_5 = 5$s. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?



Unit Exam III: Problem #4 (Fall '16)

A conducting frame of width $d = 1.6$m with a moving conducting rod is located in a uniform magnetic field $B = 3$T directed out of the plane. The rod moves at constant velocity $v = 0.4$m/s toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at times $t_2 = 2$s, $t_3 = 3$s, $t_4 = 4$s, and $t_5 = 5$s. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?



Solution:

- $\Phi_B^{(2)} = (1.6\text{m})(0.8\text{m})(3\text{T}) = 3.84\text{Wb},$
  $\mathcal{E}^{(2)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}.$
- $\Phi_B^{(3)} = (1.6\text{m})(1.2\text{m})(3\text{T}) = 5.76\text{Wb},$
  $\mathcal{E}^{(3)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}.$
- $\Phi_B^{(4)} = (1.6\text{m})(1.6\text{m})(3\text{T}) = 7.68\text{Wb},$
  $\mathcal{E}^{(4)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}.$
- $\Phi_B^{(5)} = (1.6\text{m})(2.0\text{m})(3\text{T}) = 9.60\text{Wb},$
  $\mathcal{E}^{(5)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}.$
- Clockwise current.