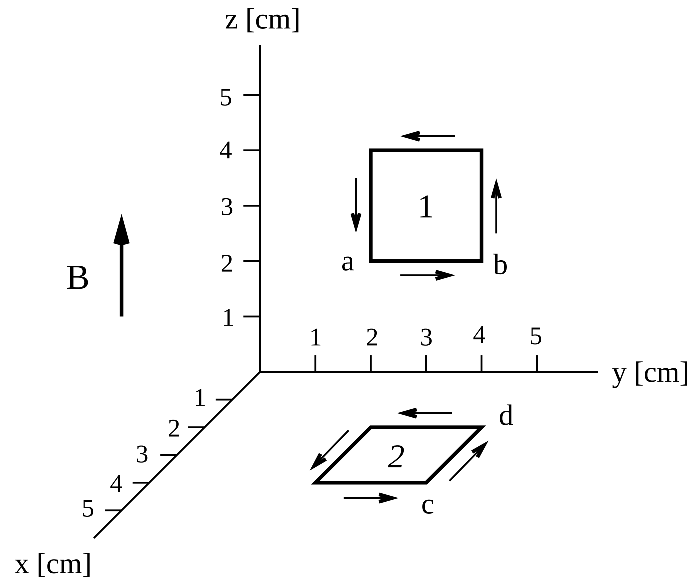
Unit Exam III: Problem #1 (Spring ’16)

Conducting squares 1 and 2, each of side 2cm, are positioned as shown. A current $I = 3$A is flowing around each square in the direction shown. A uniform magnetic field $\vec{B} = 5$mT$\hat{\mathbf{k}}$ exists in the entire region.

(a) Find the forces $\vec{F}_{ab}$ and $\vec{F}_{cd}$ acting on sides ab and cd, respectively.
(b) Find the magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of squares 1 and 2, respectively.
(c) Find the torques $\vec{\tau}_1$ and $\vec{\tau}_2$ acting on squares 1 and 2, respectively.
Remember that vectors have components or magnitude and direction.



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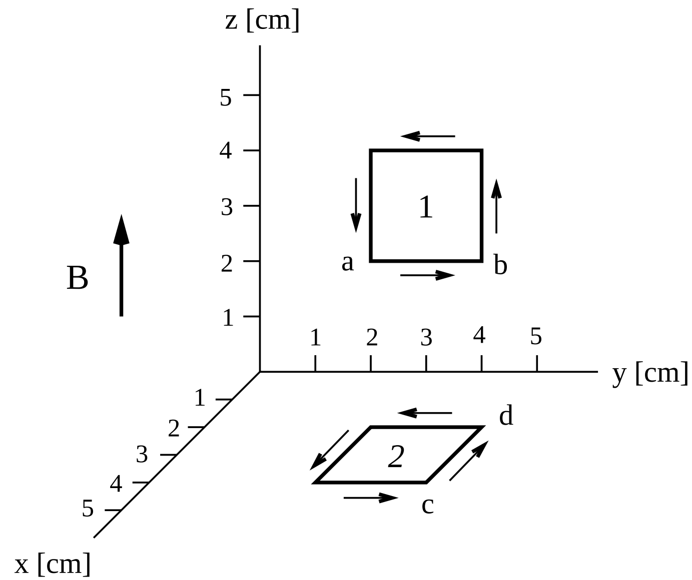


Solution:

(a) $\vec{F}_{ab} = (3\text{A})(2\text{cm}\hat{\mathbf{j}}) \times (5\text{mT}\hat{\mathbf{k}}) = 3 \times 10^{-4}\text{N}\hat{\mathbf{i}}.$

$\vec{F}_{cd} = (3\text{A})(-2\text{cm}\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{k}}) = 3 \times 10^{-4}\text{N}\hat{\mathbf{j}}.$

(b) $\vec{\mu}_1 = (2\text{cm})^2(3\text{A})\hat{\mathbf{i}} = 1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$

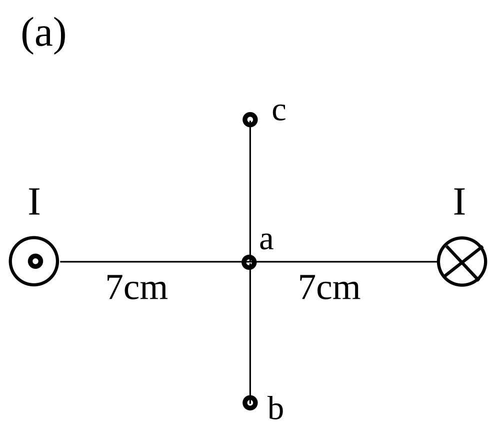
$\vec{\mu}_2 = (2\text{cm})^2(3\text{A})\hat{\mathbf{k}} = 1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{k}}.$

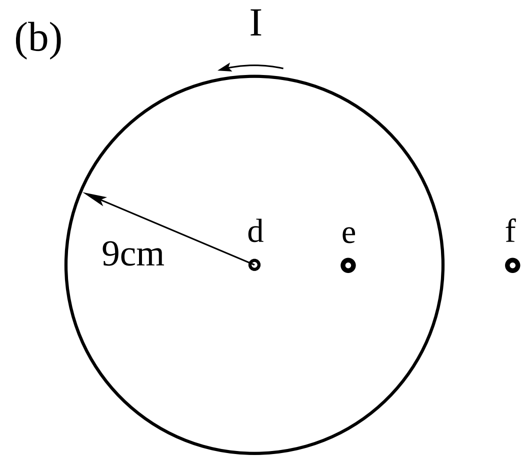
(d) $\vec{\tau}_1 = (1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{k}}) = -6 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}.$

$\vec{\tau}_2 = (1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{k}}) \times (5\text{mT}\hat{\mathbf{k}}) = 0.$

Unit Exam III: Problem #2 (Spring '16)

(a) Consider two long, straight currents $I = 3\text{mA}$ in the directions shown. Find the magnitude of the magnetic field at point a. Find the directions ($\leftarrow, \rightarrow, \uparrow, \downarrow$) of the magnetic field at points b and c.
(b) Consider a circular current $I = 3\text{mA}$ in the direction shown. Find the magnitude of the magnetic field at point d. Find the directions ($\otimes, \odot$) of the magnetic field at points e and f.





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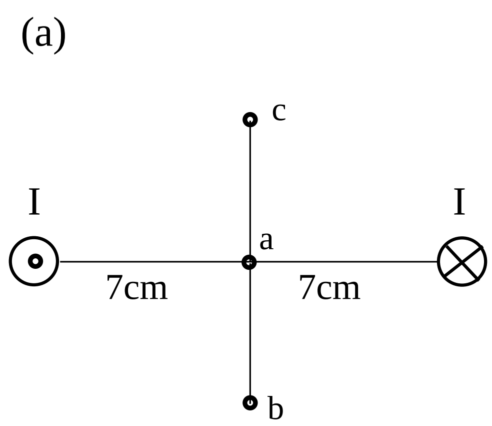
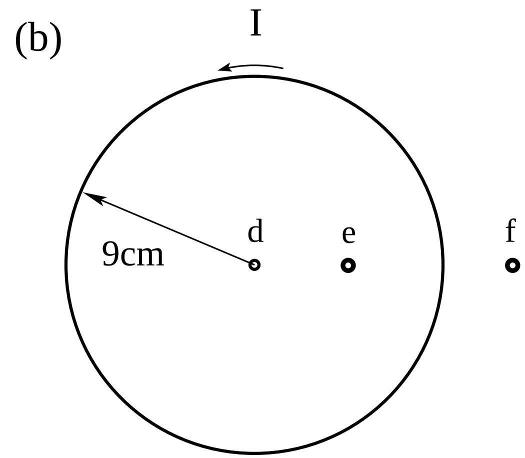




Solution:

(a) $B_a = 2\dfrac{\mu_0(3\text{mA})}{2\pi(7\text{cm})} = 1.71 \times 10^{-8}\text{T} \quad B_b \uparrow, \quad B_c \uparrow.$

(b) $B_d = \dfrac{\mu_0(3\text{mA})}{2(9\text{cm})} = 2.09 \times 10^{-8}\text{T}, \quad B_e \odot, \quad B_f \otimes.$

Unit Exam III: Problem #3 (Spring '16)

A wire shaped into a rectangular loop as shown is placed in the yz-plane. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

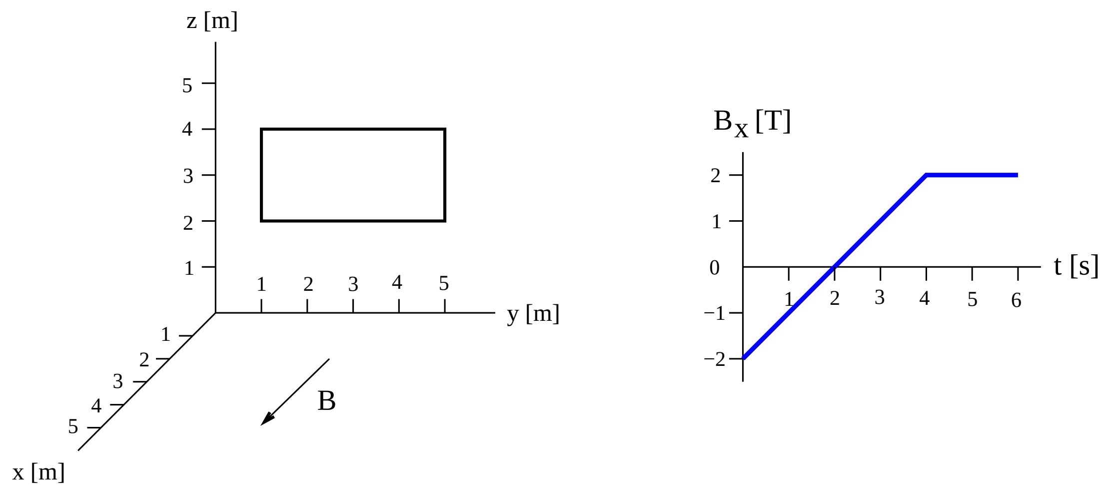
(a) Find magnitude $|\Phi_B^{(2)}|$ of the magnetic flux through the loop at time $t = 2$s.

(b) Find magnitude $|\Phi_B^{(5)}|$ of the magnetic flux through the loop at time $t = 5$s.

(c) Find magnitude $|\mathcal{E}^{(2)}|$ of the induced EMF at time $t = 2$s.

(d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5$s.

(e) Find the direction (cw/ccw) and magnitude I of the induced current at time $t = 2$s if the wire has resistance 1Ω per meter of length.



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(d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5$s.

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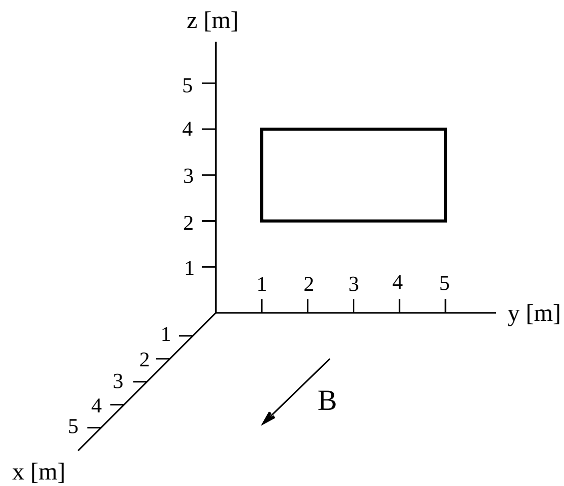

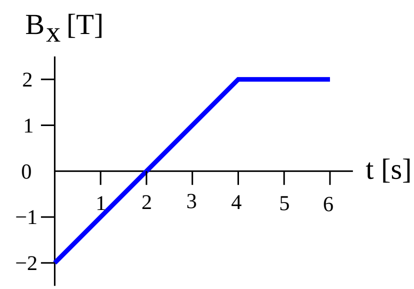

Solution:

(a) $|\Phi_B^{(2)}| = |(8\text{m}^2)(0\text{T})| = 0,$

(b) $|\Phi_B^{(5)}| = |(8\text{m}^2)(2\text{T})| = 16\,\text{Wb},$

(c) $|\mathcal{E}^{(2)}| = \left| A\dfrac{dB}{dt} \right| = |(8\text{m}^2)(1\text{T/s}) = 8\text{V}$

(d) $|\mathcal{E}^{(5)}| = \left| A\dfrac{dB}{dt} \right| = |(8\text{m}^2)(0\text{T/s}) = 0$

(e) $I^{(2)} = \dfrac{8\text{V}}{12\Omega} = 0.667\text{A}.$ (cw).