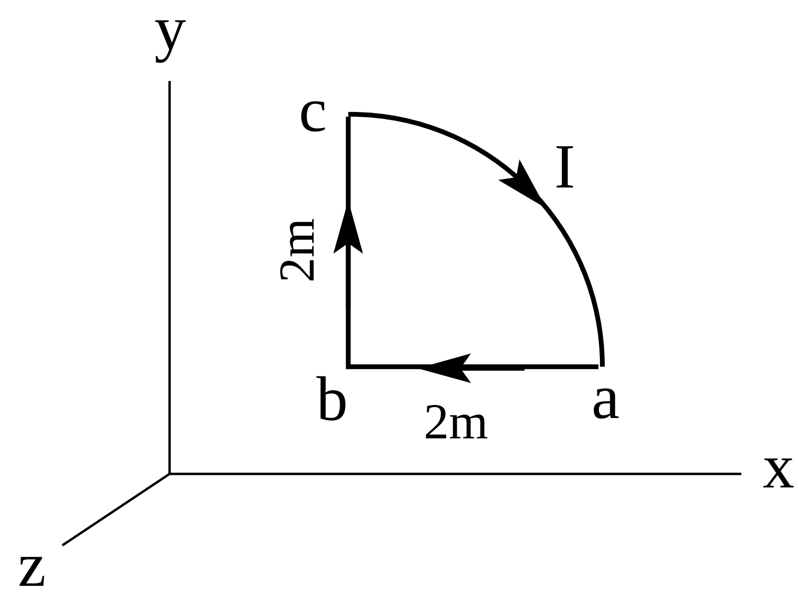
Unit Exam III: Problem #1 (Fall ’15)

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side (i) ab, (ii) bc of the loop.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



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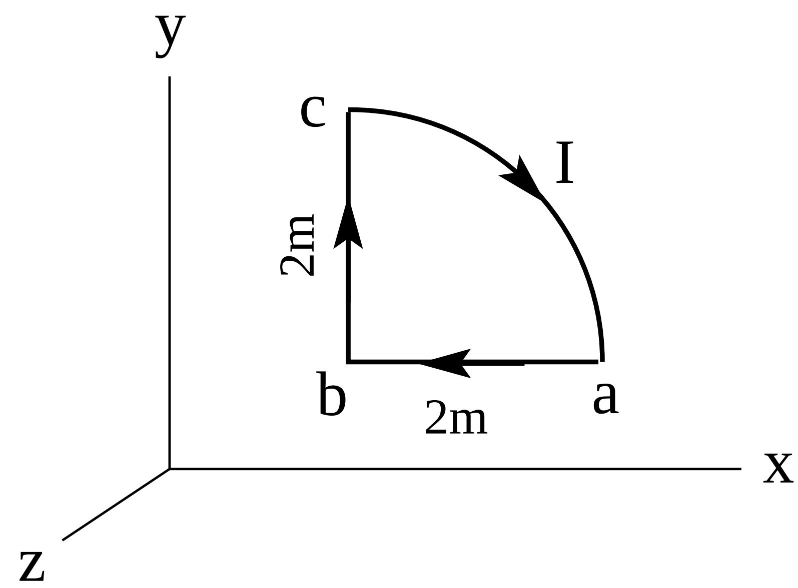

Solution:

(ia) $\vec{\mu} = (4\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -12.6\mathrm{Am}^2\hat{k}.$

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.
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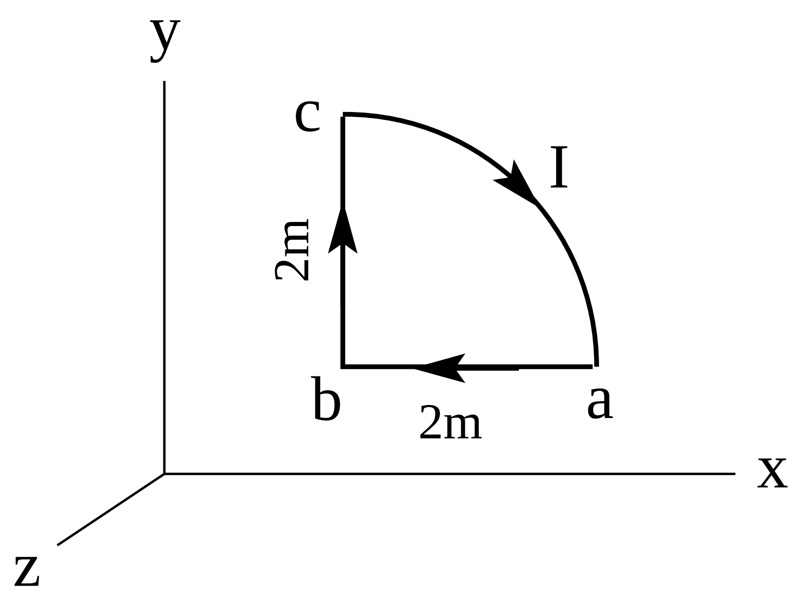

Solution:

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(ib) $\vec{F}_{ab} = (4\mathrm{A})(-2\mathrm{m}\hat{i}) \times (5\mathrm{T}\hat{j}) = -40\mathrm{N}\hat{k}.$

Unit Exam III: Problem #1 (Fall ’15)

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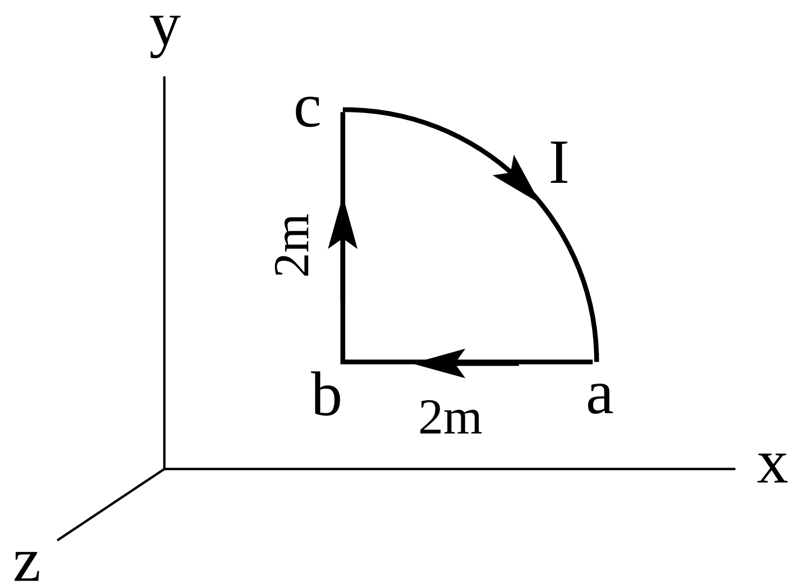

Solution:

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(ic) $\vec{\tau} = (-12.6\mathrm{Am}^2\hat{k}) \times (5\mathrm{T}\hat{j}) = 63.0\mathrm{Nm}\hat{i}$

Unit Exam III: Problem #1 (Fall '15)

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.
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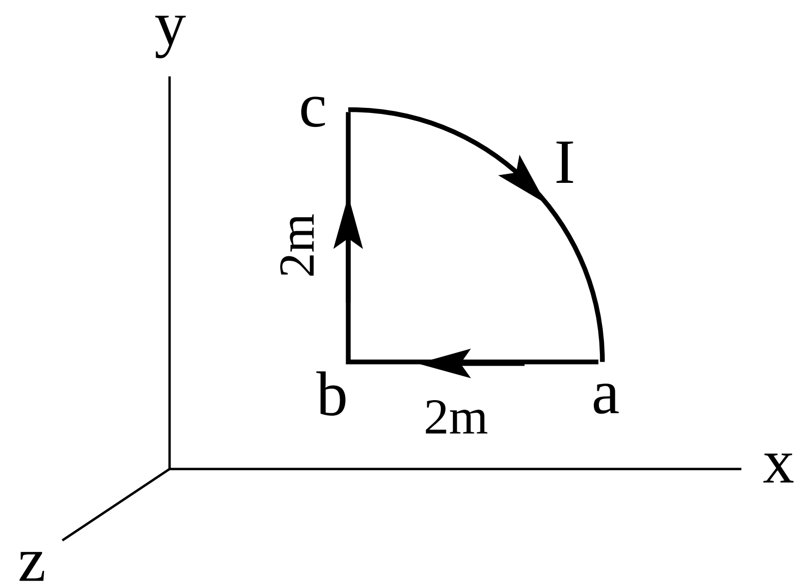

Solution:

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(ic) $\vec{\tau} = (-12.6\mathrm{Am}^2\hat{k}) \times (5\mathrm{T}\hat{j}) = 63.0\mathrm{Nm}\hat{i}$

(iia) $\vec{\mu} = (3\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -9.42\mathrm{Am}^2\hat{k}.$

Unit Exam III: Problem #1 (Fall '15)

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.
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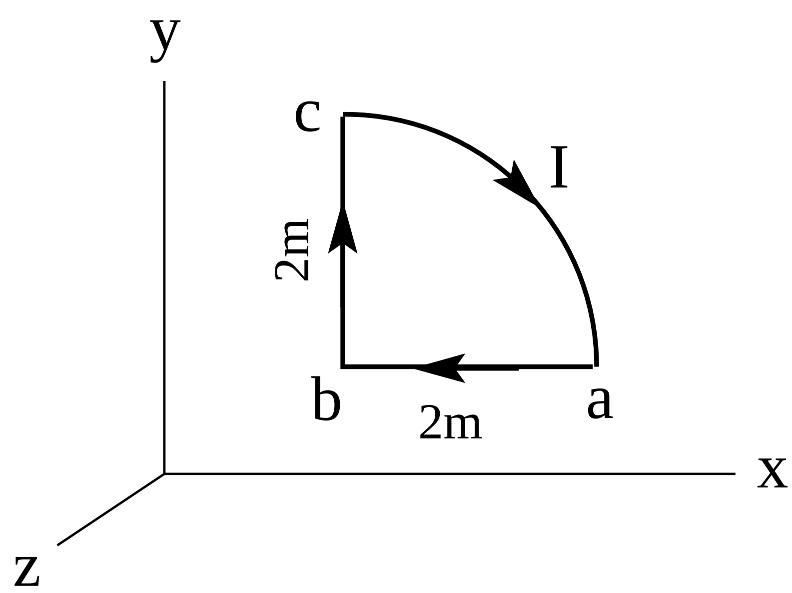

Solution:

(ia) $\vec{\mu} = (4\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -12.6\mathrm{Am}^2\hat{k}$.

(ib) $\vec{F}_{ab} = (4\mathrm{A})(-2\mathrm{m}\hat{i}) \times (5\mathrm{T}\hat{j}) = -40\mathrm{N}\hat{k}$.

(ic) $\vec{\tau} = (-12.6\mathrm{Am}^2\hat{k}) \times (5\mathrm{T}\hat{j}) = 63.0\mathrm{Nm}\hat{i}$

(iia) $\vec{\mu} = (3\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -9.42\mathrm{Am}^2\hat{k}$.

(iib) $\vec{F}_{bc} = (3\mathrm{A})(2\mathrm{m}\hat{j}) \times (-6\mathrm{T}\hat{i}) = 36\mathrm{N}\hat{k}$.

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.
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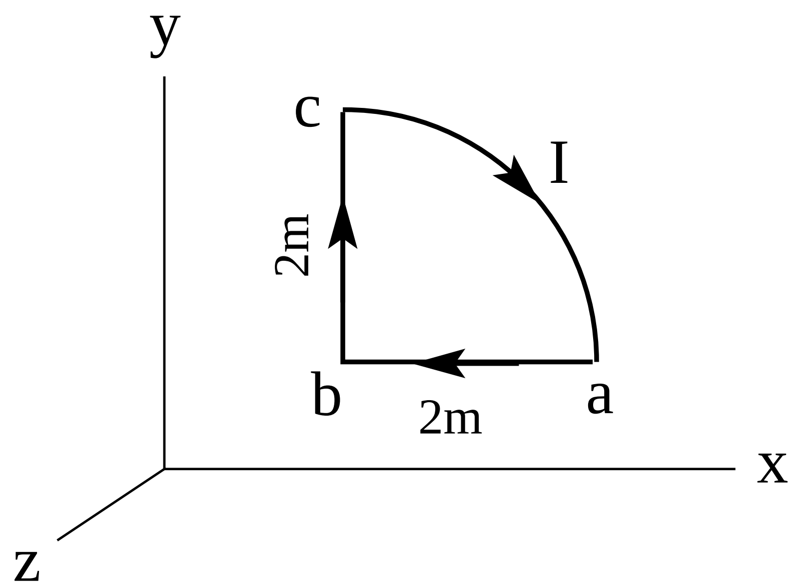

Solution:

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(ib) $\vec{F}_{ab} = (4\mathrm{A})(-2\mathrm{m}\hat{i}) \times (5\mathrm{T}\hat{j}) = -40\mathrm{N}\hat{k}$.

(ic) $\vec{\tau} = (-12.6\mathrm{Am}^2\hat{k}) \times (5\mathrm{T}\hat{j}) = 63.0\mathrm{Nm}\hat{i}$

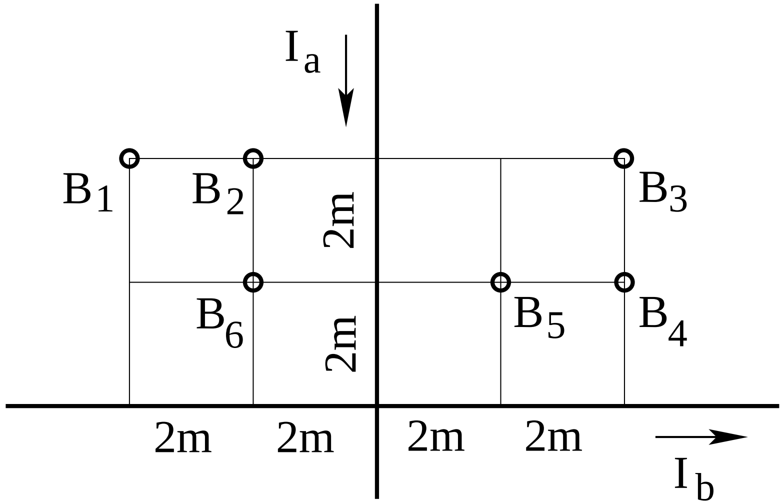
(iia) $\vec{\mu} = (3\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -9.42\mathrm{Am}^2\hat{k}$.

(iib) $\vec{F}_{bc} = (3\mathrm{A})(2\mathrm{m}\hat{j}) \times (-6\mathrm{T}\hat{i}) = 36\mathrm{N}\hat{k}$.

(iic) $\vec{\tau} = (-9.42\mathrm{Am}^2\hat{k}) \times (-6\mathrm{T}\hat{i}) = 56.5\mathrm{Nm}\hat{j}$

Unit Exam III: Problem #2 (Fall '15)

Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall '15)

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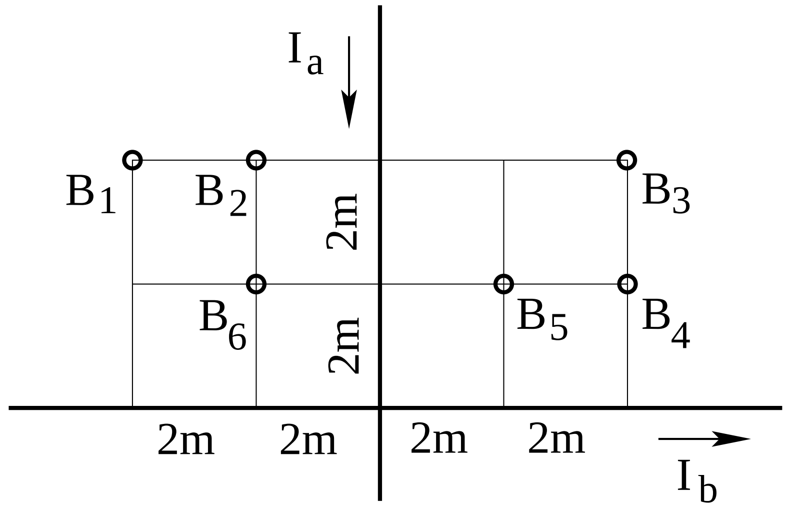
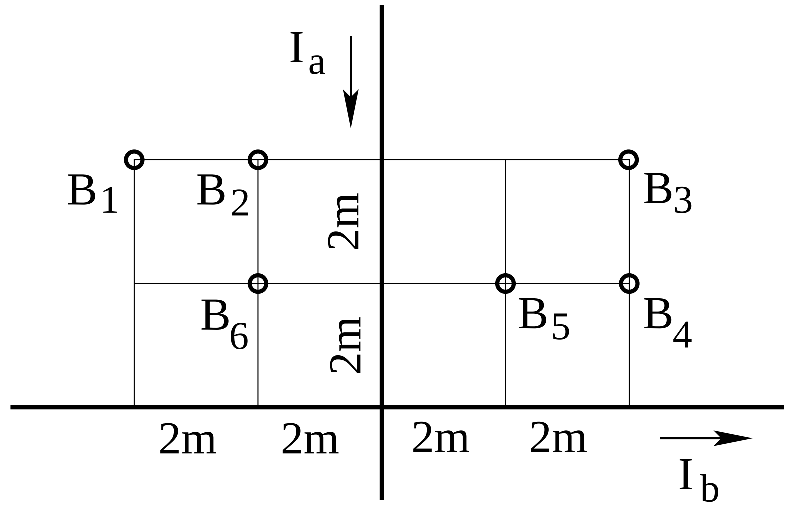


Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).

Unit Exam III: Problem #2 (Fall '15)

Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.

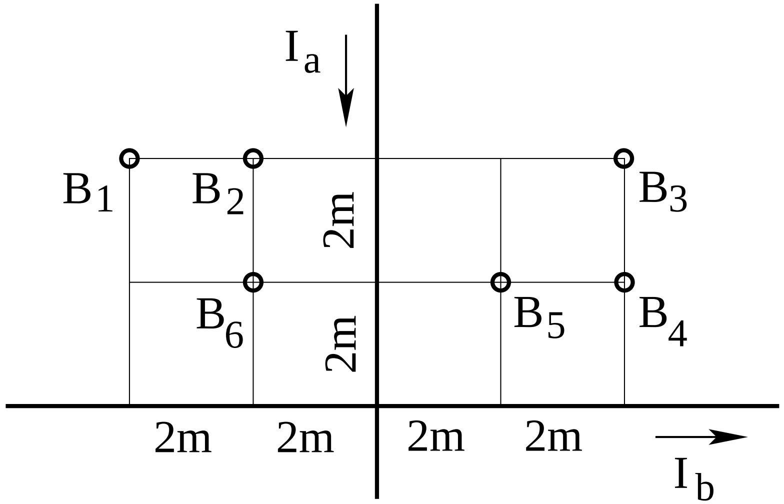


Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = -0.3\mu\text{T}$ (into plane).

Unit Exam III: Problem #2 (Fall '15)

Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.



Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = -0.3\mu\text{T}$ (into plane).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = +0.6\mu\text{T}$ (out of plane).

Unit Exam III: Problem #2 (Fall ’15)

Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.

Solution:

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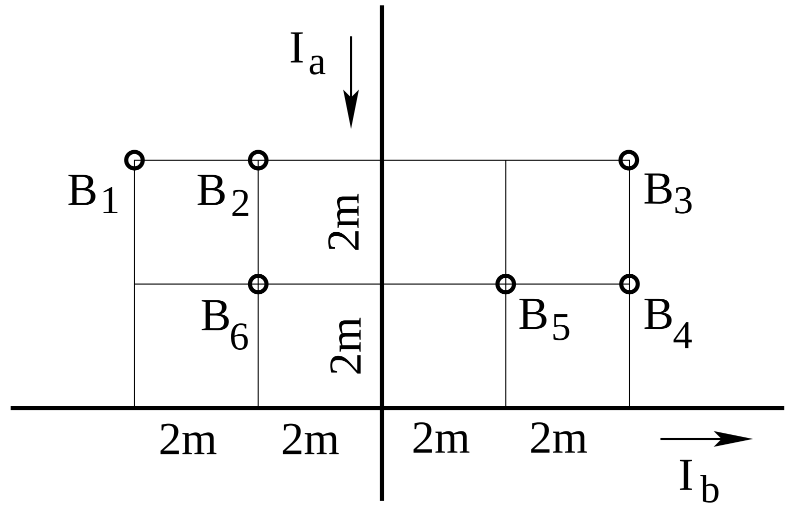
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = 0.9\mu\text{T}$ (out of plane).



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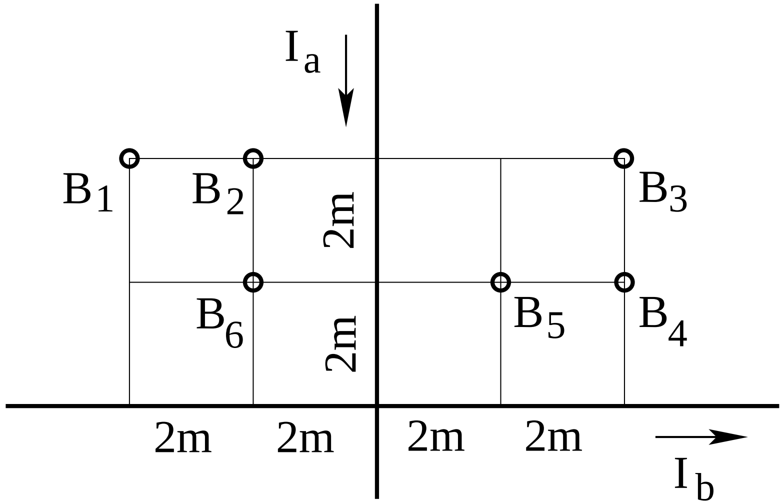

Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).
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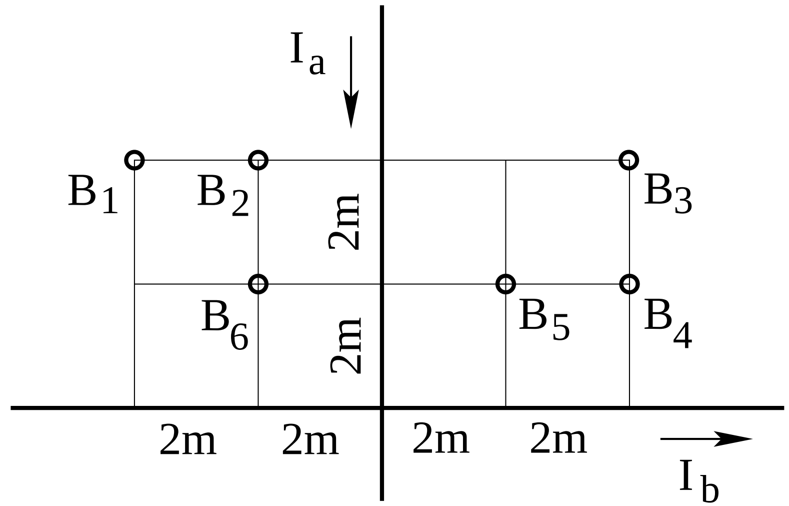
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = +0.6\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = 0.9\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{2\text{m}}\right) = 1.2\mu\text{T}$ (out of plane).

Unit Exam III: Problem #2 (Fall '15)

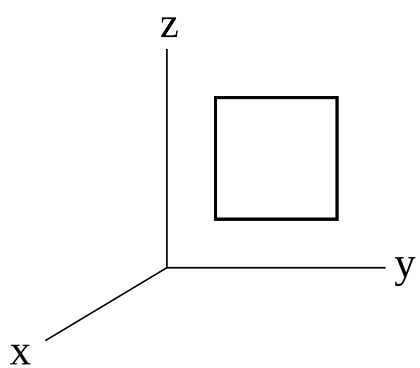
Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.

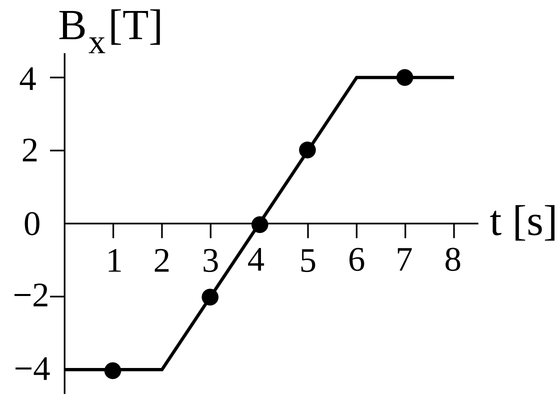


Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = -0.3\mu\text{T}$ (into plane).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = +0.6\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = 0.9\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{2\text{m}}\right) = 1.2\mu\text{T}$ (out of plane).
- $B_6 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = 0$ (no direction).

A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.
(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .
(b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
(c) Find the direction (cw, ccw, zero) of the induced current at the above times.





Solution:

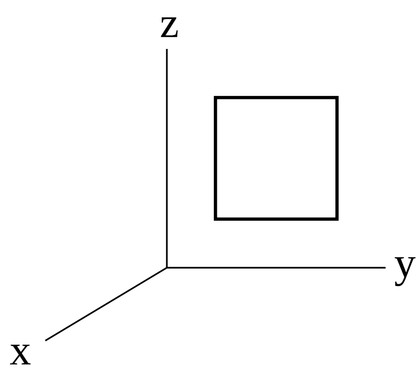
Unit Exam III: Problem #3 (Fall '15)

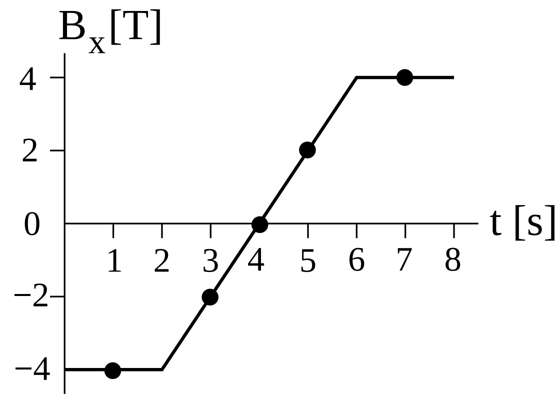
A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.

(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .

(b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.

(c) Find the direction (cw, ccw, zero) of the induced current at the above times.





Solution:

(ia) $|\Phi_B^{(1)}| = (1.44\text{m}^2)(4\text{T}) = 5.76\,\text{Wb}$

$|\Phi_B^{(3)}| = (1.44\text{m}^2)(2\text{T}) = 2.88\,\text{Wb}$

$|\Phi_B^{(4)}| = (1.44\text{m}^2)(0\text{T}) = 0$

(ib) $\mathcal{E}^{(1)} = (1.44\text{m}^2)(0\text{T/s}) = 0$

$\mathcal{E}^{(3)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$

$\mathcal{E}^{(4)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$

(ic) zero, cw, cw

Unit Exam III: Problem #3 (Fall '15)

A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.
(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .
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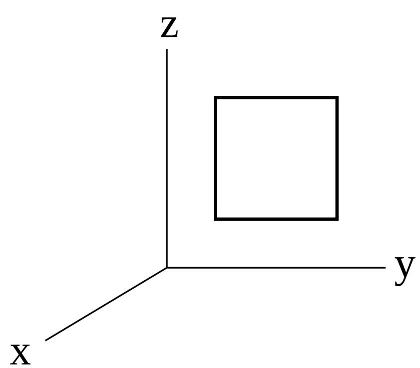

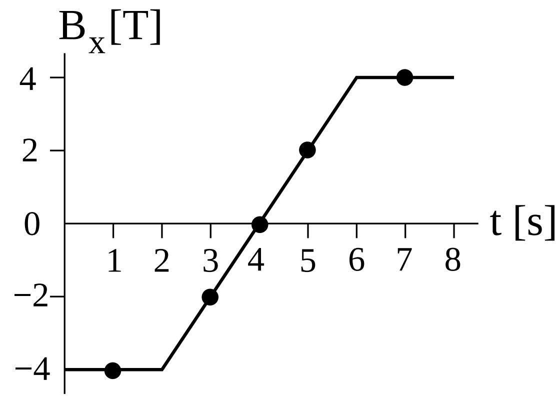

Solution:

(iia) $|\Phi_B^{(4)}| = (1.69\mathrm{m}^2)(0\mathrm{T}) = 0$

$|\Phi_B^{(5)}| = (1.69\mathrm{m}^2)(2\mathrm{T}) = 3.38\,\mathrm{Wb}$

$|\Phi_B^{(7)}| = (1.69\mathrm{m}^2)(4\mathrm{T}) = 6.76\,\mathrm{Wb}$

(iib) $\mathcal{E}^{(4)} = (1.69\mathrm{m}^2)(2\mathrm{T/s}) = 3.38\mathrm{V}$

$\mathcal{E}^{(5)} = (1.69\mathrm{m}^2)(2\mathrm{T/s}) = 3.38\mathrm{V}$

$\mathcal{E}^{(7)} = (1.69\mathrm{m}^2)(0\mathrm{T/s}) = 0$

(iic) cw, cw, zero