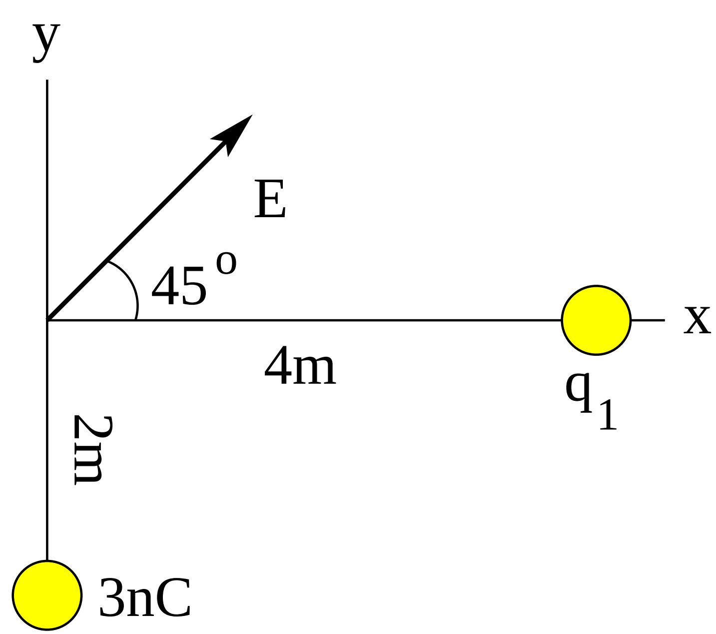
Intermediate Exam I: Problem #1 (Spring '05)

The electric field $\vec{E}$ generated by the two point charges, 3nC and q_1 (unknown), has the direction shown.

(a) Find the magnitude of $\vec{E}$.

(b) Find the value of q_1.



Solution:

(a) $E_y = k\dfrac{3\text{nC}}{(2\text{m})^2} = 6.75\text{N/C},$

$E_x = E_y,$

$E = \sqrt{E_x^2 + E_y^2} = 9.55\text{N/C}.$

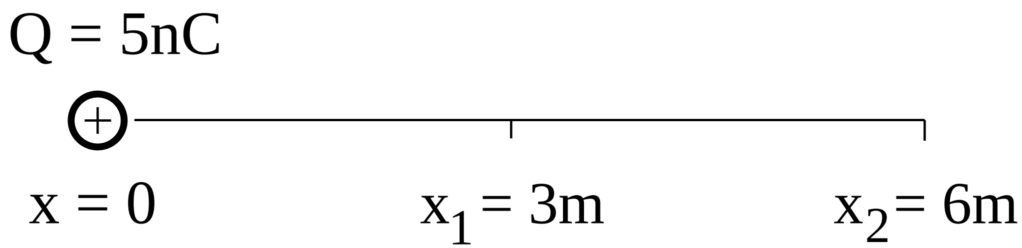
(b) $E_x = k\dfrac{(-q_1)}{(4\text{m})^2},$

$$q_1 = -\frac{(6.75\text{N/C})(16\text{m}^2)}{k} = -12\text{nC}.$$

Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3\text{m}$
and the electric potiential V_2 at position $x_2 = 6\text{m}$.

(b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



Solution:

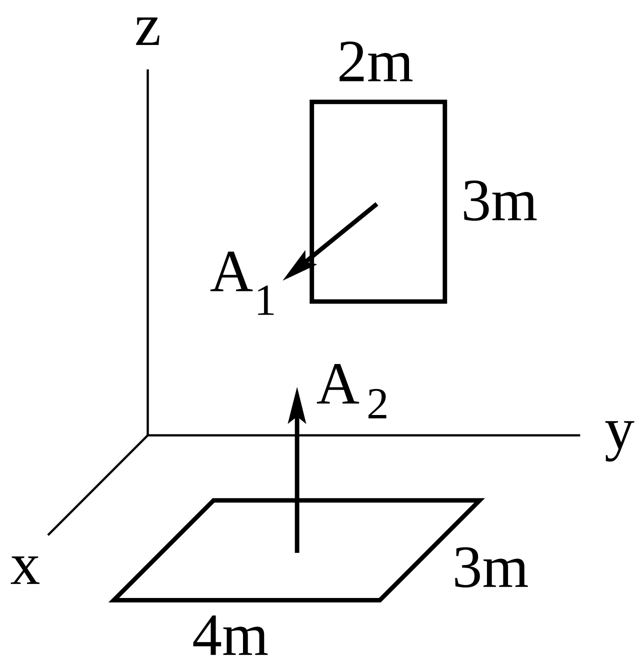
(a) $V_1 = k\dfrac{Q}{x_1} = 15\text{V}, \quad V_2 = k\dfrac{Q}{x_2} = 7.5\text{V}.$

(b) $\Delta U = q(V_2 - V_1) = (4\text{nC})(-7.5\text{V}) = -30\text{nJ} \quad \Rightarrow \; \Delta K = -\Delta U = 30\text{nJ}.$

$$\Delta K = K_2 = \frac{1}{2}mv_2^2 \quad \Rightarrow \; v_2 = \sqrt{\frac{2K_2}{m}} = 200\text{m/s}.$$

Intermediate Exam I: Problem #3 (Spring ’05)

Consider two plane surfaces with area vectors $\vec{A}_1$ (pointing in positive x-direction) and $\vec{A}_2$ (pointing in positive z-direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{i} + 7\hat{j} - 3\hat{k})$N/C.



(a) Find the electric flux $\Phi_E^{(1)}$ through area A_1.

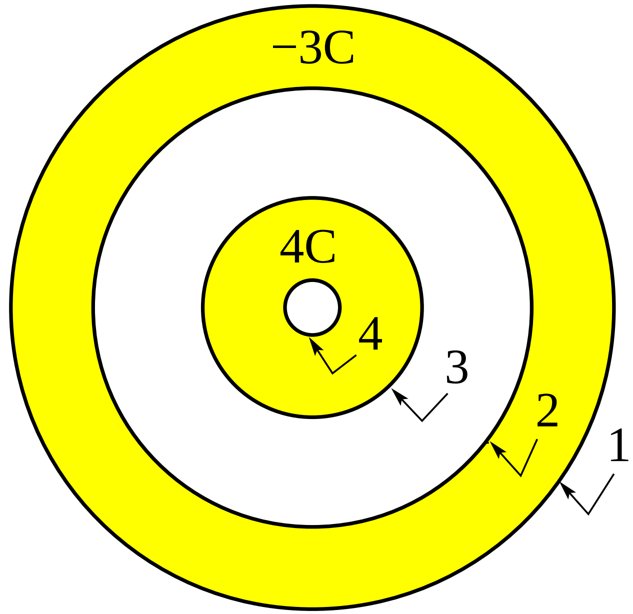
(b) Find the electric flux $\Phi_E^{(2)}$ through area A_2.

Solution:

(a) $\vec{A}_1 = 6\hat{i}\,\mathrm{m}^2$,
$\Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (2\mathrm{N/C})(6\mathrm{m}^2) = 12\mathrm{Nm}^2/\mathrm{C}.$

(b) $\vec{A}_2 = 12\hat{k}\,\mathrm{m}^2$,
$\Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (-3\mathrm{N/C})(12\mathrm{m}^2) = -36\mathrm{Nm}^2/\mathrm{C}.$

Consider two concentric conducting spherical shells. The total electric charge on the inner shell is 4C and the total electric charge on the outer shell is -3C. Find the electric charges q_1, q_2, q_3, q_4 on each surface of both shells as identified in the figure.



Solution:

Start with the innermost surface.
Note that any excess charge is located at the surface of a conductor.
Note also that the electric field inside a conductor at equilibrium vanishes.

- Gauss’ law predicts $q_4 = 0$.
- Charge conservation then predicts $q_3 + q_4 = 4$C. Hence $q_3 = 4$C.
- Gauss’ law predicts $q_2 = -(q_3 + q_4) = -4$C.
- Charge conservation then predicts $q_1 + q_2 = -3$C. Hence $q_1 = +1$C.