RC Circuit: Charging the Capacitor

- Loop rule: $\mathcal{E} - IR - \dfrac{Q}{C} = 0$
- Differential equation: $R\dfrac{dQ}{dt} + \dfrac{Q}{C} = \mathcal{E} \quad \Rightarrow \quad \dfrac{dQ}{dt} = \dfrac{\mathcal{E}C - Q}{RC}$

$$\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right) = \frac{t}{RC} \quad \Rightarrow \quad \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}$$

- Charge on capacitor: $Q(t) = \mathcal{E}C\left[1 - e^{-t/RC}\right]$
- Current through resistor: $I(t) \equiv \dfrac{dQ}{dt} = \dfrac{\mathcal{E}}{R}\, e^{-t/RC}$