A battery providing an emf $\mathcal{E}$ with internal resistance r is connected to an external resistor of resistance R as shown.

For what value of R does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \quad I = \dfrac{\mathcal{E}}{R + r}$

- Power delivered to external resistor: $P = I^2 R = \dfrac{\mathcal{E}^2 R}{(R + r)^2}$

- Condition for maximum power: $\dfrac{dP}{dR} = 0 \quad \Rightarrow \quad R = r$