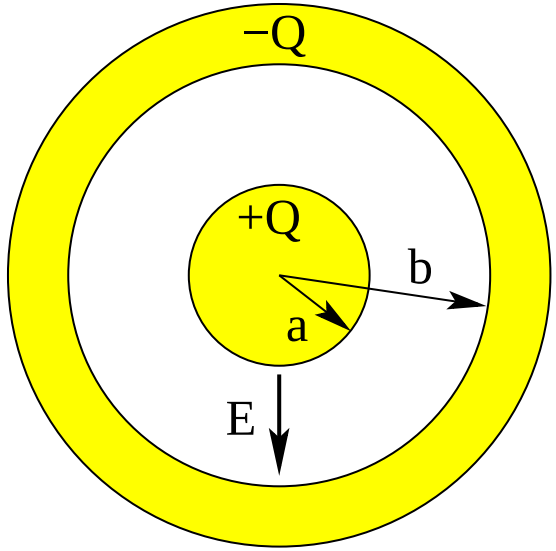
Integrating Energy Density in Spherical Capacitor

- Electric field: $E(r) = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{r^2}$
- Voltage: $V = \dfrac{Q}{4\pi\epsilon_0}\dfrac{b-a}{ab} = \dfrac{Q}{4\pi\epsilon_0}\left[\dfrac{1}{a} - \dfrac{1}{b}\right]$
- Energy density: $u_E(r) = \dfrac{1}{2}\epsilon_0 E^2(r)$



- Energy stored in capacitor: $U = \displaystyle\int_a^b u_E(r)(4\pi r^2)dr$
- $\Rightarrow\ U = \displaystyle\int_a^b \frac{1}{2}\epsilon_0 \frac{Q^2}{(4\pi\epsilon_0)^2}\frac{1}{r^4}(4\pi r^2)dr$
- $\Rightarrow\ U = \dfrac{1}{2}\dfrac{Q^2}{4\pi\epsilon_0}\displaystyle\int_a^b \frac{1}{r^2}dr = \frac{1}{2}\frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{1}{2}QV$