Spherical Capacitor

Conducting sphere of radius a surrounded concentrically by conducting spherical shell of inner radius b.

- Q: magnitude of charge on each sphere
- Electric field between spheres: use Gauss' law

  $$E[4\pi r^2] = \frac{Q}{\epsilon_0} \;\Rightarrow\; E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

- Electric potential between spheres: use $V(a) = 0$

  $$V(r) = -\int_a^r E(r)dr = -\frac{Q}{4\pi\epsilon_0}\int_a^r \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{a}\right]$$

- Voltage between spheres:

  $$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{4\pi\epsilon_0}\frac{b-a}{ab}$$

- Capacitance for spherical geometry:

  $$C \equiv \frac{Q}{V} = 4\pi\epsilon_0\frac{ab}{b-a}$$