Cylindrical Capacitor

Conducting cylinder of radius a and length L surrounded concentrically by conducting cylindrical shell of inner radius b and equal length.

- Assumption: $L \gg b$.
- λ: charge per unit length (magnitude) on each cylinder
- $Q = \lambda L$: magnitude of charge on each cylinder
- Electric field between cylinders: use Gauss' law

  $$E[2\pi r L] = \frac{\lambda L}{\epsilon_0} \;\Rightarrow\; E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

- Electric potential between cylinders: use $V(a) = 0$

  $$V(r) = -\int_a^r E(r)dr = -\frac{\lambda}{2\pi\epsilon_0}\int_a^r \frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{r}{a}$$

- Voltage between cylinders:

  $$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{2\pi\epsilon_0 L}\ln\frac{b}{a}$$

- Capacitance for cylindrical geometry:

  $$C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$