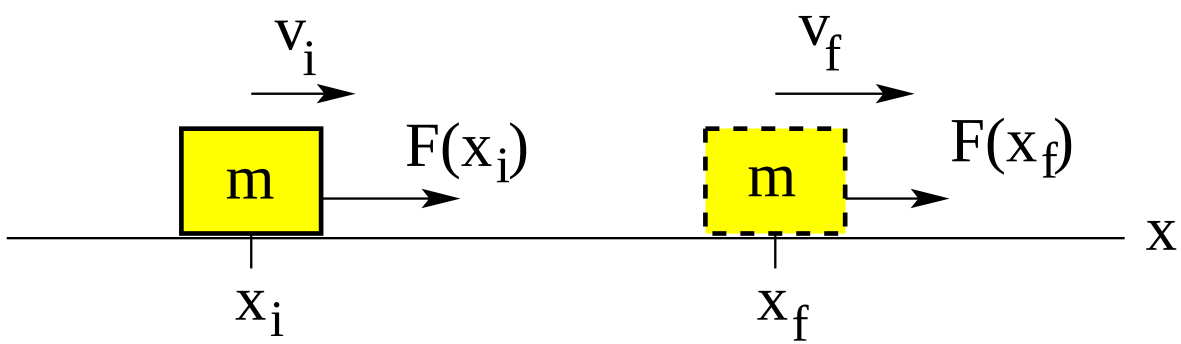
Work and Energy

Consider a block of mass m moving along the x-axis.

- Conservative force acting on block: $F = F(x)$
- Work done by $F(x)$ on block: $W_{if} = \int_{x_i}^{x_f} F(x)dx$
- Kinetic energy of block: $K = \frac{1}{2}mv^2$
- Potential energy of block: $U(x) = -\int_{x_0}^{x} F(x)dx \;\Rightarrow\; F(x) = -\frac{dU}{dx}$
- Transformation of energy: $\Delta K \equiv K_f - K_i,\ \Delta U \equiv U_f - U_i$
- Total mechanical energy: $E = K + U = \text{const} \;\Rightarrow\; \Delta K + \Delta U = 0$
- Work-energy relation: $W_{if} = \Delta K = -\Delta U$



Conservative Forces in Mechanics

Conservative forces familiar from mechanics:

- Elastic force: $F(x) = -kx \quad \Rightarrow U(x) = -\int_{x_0}^{x}(-kx)dx = \frac{1}{2}kx^2 \qquad (x_0 = 0).$
- Gravitational force (locally): $F(y) = -mg$

$$\Rightarrow U(y) = -\int_{y_0}^{y}(-mg)dy = mgy \qquad (y_0 = 0).$$

- Gravitational force (globally): $F(r) = -G\frac{mm_E}{r^2}$

$$\Rightarrow U(r) = -\int_{r_0}^{r}\left(-G\frac{mm_E}{r^2}\right)dr = -G\frac{mm_E}{r} \qquad (r_0 = \infty).$$

Potential energy depends on integration constant.

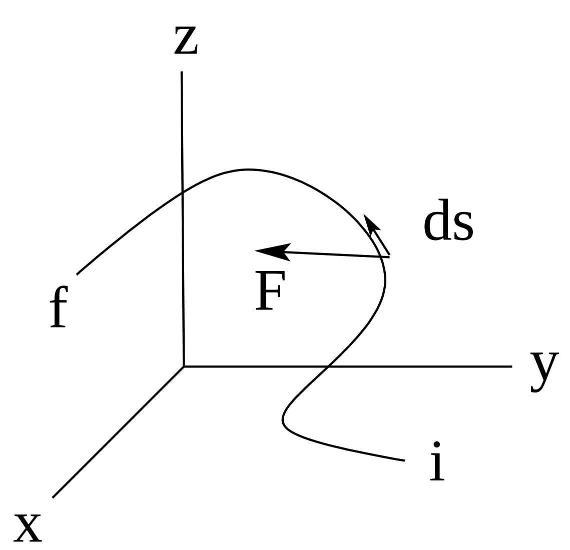
Integration constant determines reference position where $U = 0$:
$x = x_0,\ y = y_0,\ r = r_0.$

Work and Potential Energy in 3D Space

Consider a particle acted on by a force $\vec{F}$ as it moves along a specific path in 3D space.

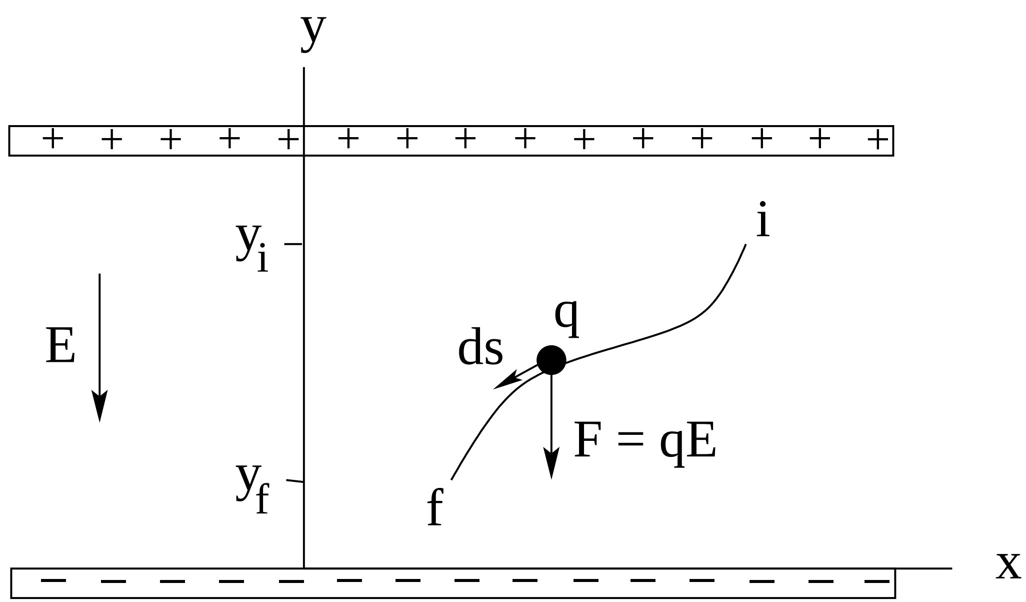
- Force: $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
- Work: $W_{if} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$
- Potential energy: $U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{s} = -\int_{x_0}^{x} F_x dx - \int_{y_0}^{y} F_y dy - \int_{z_0}^{z} F_z dz$

Note: The work done by a conservative force is path-independent.



Potential Energy of Charged Particle in Uniform Electric Field

- Electrostatic force: $\vec{F} = -qE\hat{j}$ (conservative)
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j}$
- Work: $W_{if} = \int_i^f \vec{F} \cdot d\vec{s} = \int_{y_i}^{y_f} (-qE)dy = -qE(y_f - y_i)$
- Potential energy: $U = -\int_0^y (-qE)dy = qEy$
- Electric potential: $V(y) = Ey$

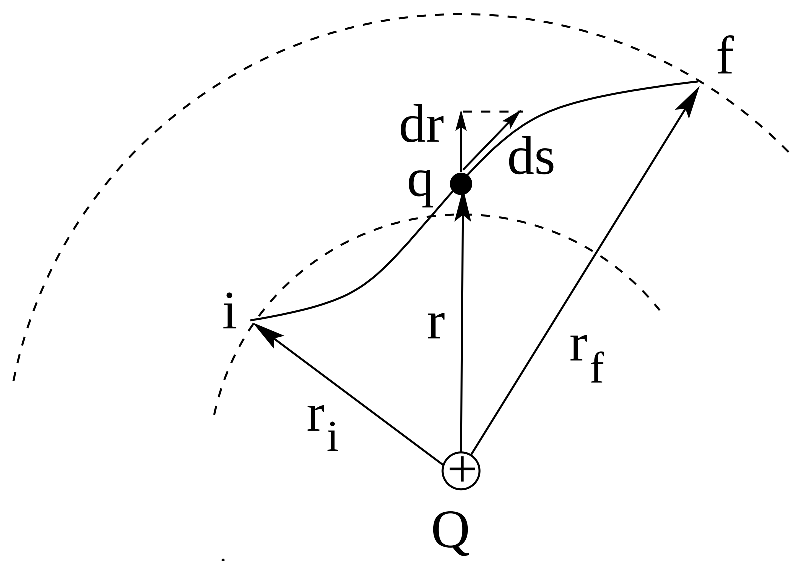


Potential Energy of Charged Particle in Coulomb Field

- Electrostatic force: $\vec{F} = \frac{kqQ}{r^2}\hat{r}$ (conservative)
- Displacement: $d\vec{s} = d\vec{r} + d\vec{s}_\perp, \quad d\vec{r} = dr\hat{r}$
- Work: $W_{if} = \int_i^f \vec{F}\cdot d\vec{s} = kqQ\int_i^f \frac{\hat{r}\cdot d\vec{s}}{r^2} = kqQ\int_{r_i}^{r_f}\frac{dr}{r^2}$

$$= kqQ\left[-\frac{1}{r}\right]_{r_i}^{r_f} = -kqQ\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

- Potential energy: $U = -\int_\infty^r F dr = -kqQ\int_\infty^r \frac{dr}{r^2} = k\frac{qQ}{r}$
- Electric potential: $V(r) = \frac{kQ}{r}$



Attributes of Space and of Charged Particles

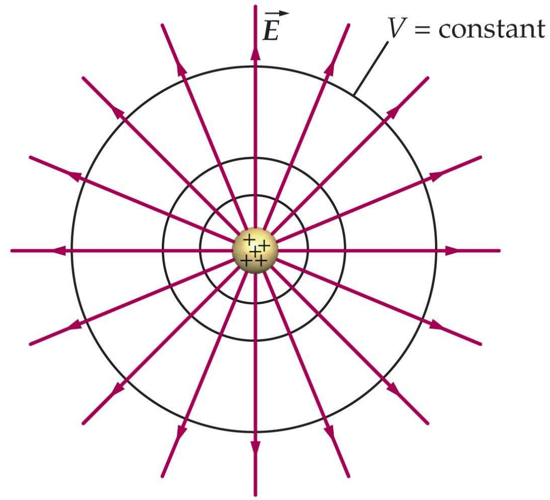
	planar source	point source	SI unit
electric field	$\vec{E} = E_x \hat{i}$	$\vec{E} = \frac{kQ}{r^2}\hat{r}$	[N/C]=[V/m]
electric potential	$V = -E_x x$	$V = \frac{kQ}{r}$	[V]=[J/C]
electric force	$\vec{F} = q\vec{E} = qE_x \hat{i}$	$\vec{F} = q\vec{E} = \frac{kQq}{r^2}\hat{r}$	[N]
electric potential energy	$U = qV = -qE_x x$	$U = qV = \frac{kQq}{r}$	[J]

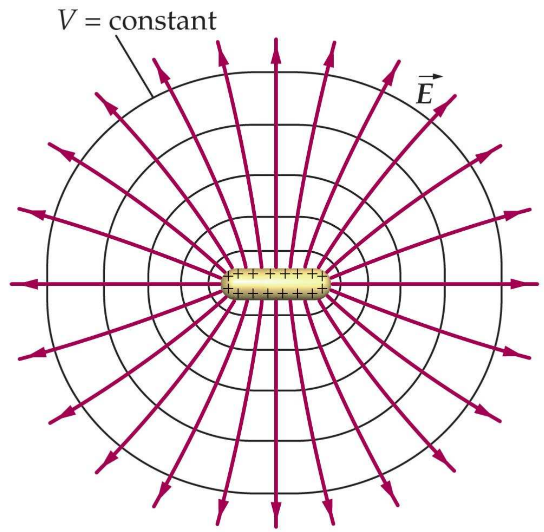
Electric field $\vec{E}$ is present at points in space.
Points in space are at electric potential V.

Charged particles experience electric force $\vec{F} = q\vec{E}$.
Charged particles have electric potential energy $U = qV$.

Equipotential Surfaces and Field Lines

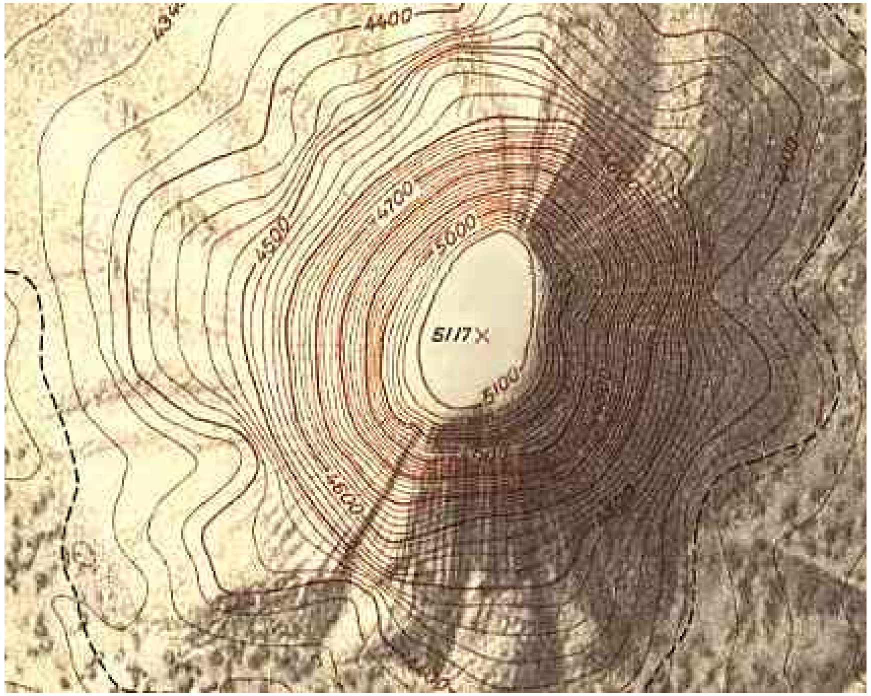
- Definition: $V(\vec{r}) = $ const on equipotential surface.
- Potential energy $U(\vec{r}) = $ const for point charge q on equipotential surface.
- The surface of a conductor at equilibrium is an equipotential surface.
- Electric field vectors $\vec{E}(\vec{r})$ (tangents to field lines) are perpendicular to equipotential surface.
- Electrostatic force $\vec{F} = q\vec{E}(\vec{r})$ does zero work on point charge q moving on equipotential surface.
- The electric field $\vec{E}(\vec{r})$ exerts a force on a positive (negative) point charge q in the direction of steepest potential drop (rise).
- When a positive (negative) point charge q moves from a region of high potential to a region of low potential, the electric field does positive (negative) work on it. In the process, the potential energy decreases (increases).



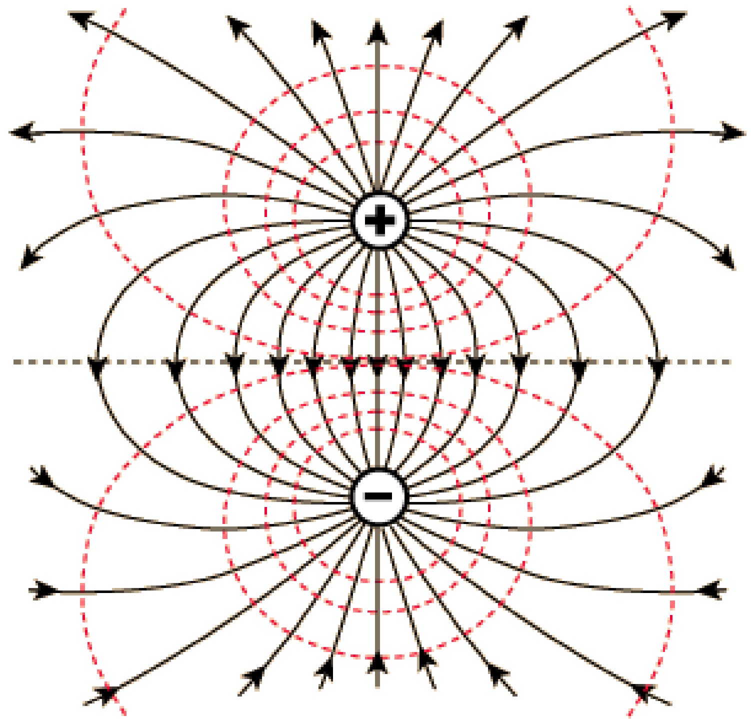


Topographic Maps

Gravitation

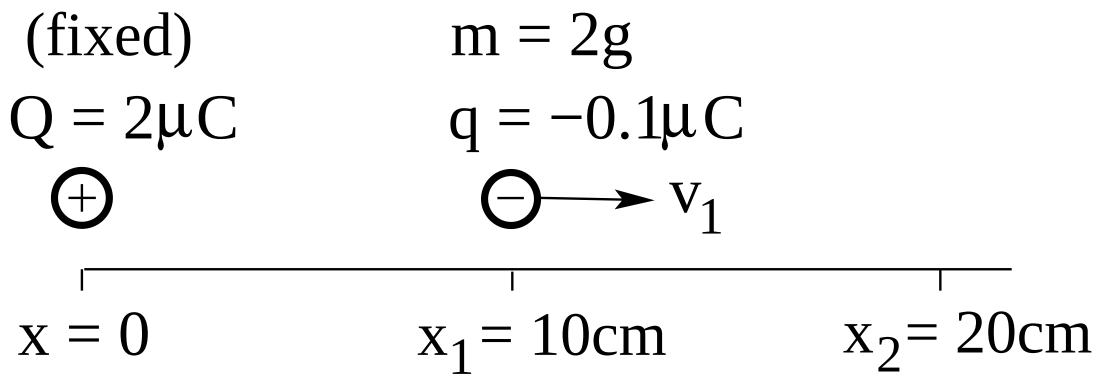


Electricity



Electric Potential and Potential Energy: Application (1)

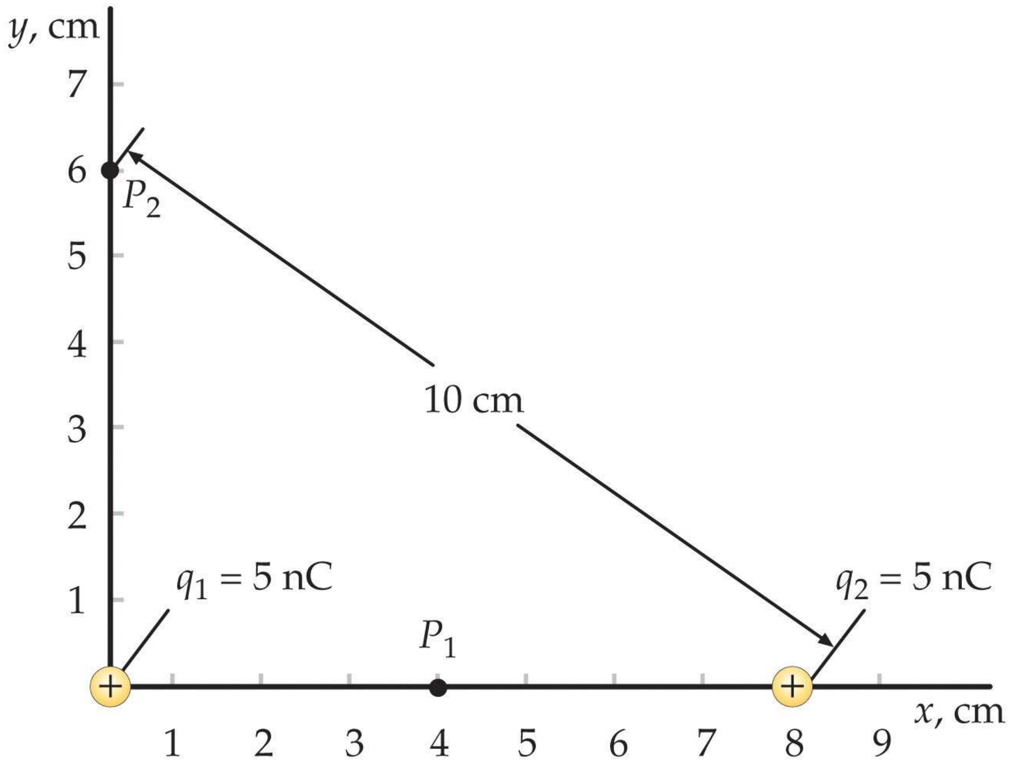
Consider a point charge $Q = 2\mu$C fixed at position $x = 0$. A particle with mass $m = 2$g and charge $q = -0.1\mu C$ is launched at position $x_1 = 10$cm with velocity $v_1 = 12$m/s.



- Find the velocity v_2 of the particle when it is at position $x_2 = 20$cm.

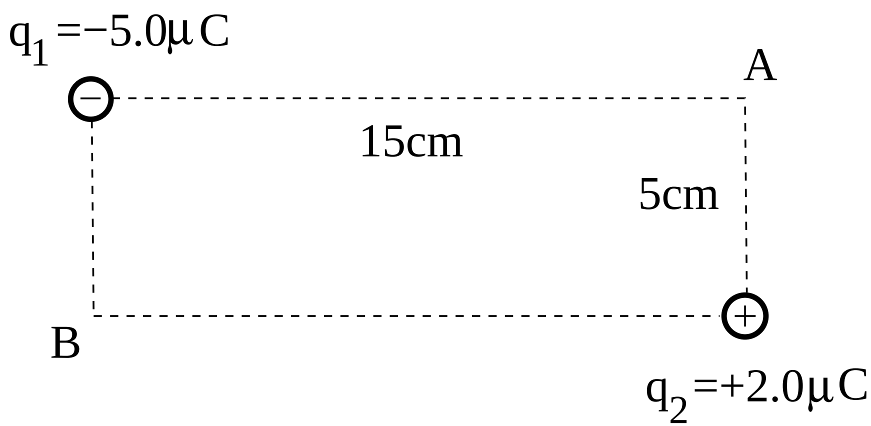
Electric Potential and Potential Energy: Application (2)

- Electric potential at point P_1: $V = \dfrac{kq_1}{0.04\mathrm{m}} + \dfrac{kq_2}{0.04\mathrm{m}} = 1125\mathrm{V} + 1125\mathrm{V} = 2250\mathrm{V}.$
- Electric potential at point P_2: $V = \dfrac{kq_1}{0.06\mathrm{m}} + \dfrac{kq_2}{0.10\mathrm{m}} = 750\mathrm{V} + 450\mathrm{V} = 1200\mathrm{V}.$



Electric Potential and Potential Energy: Application (3)

Point charges $q_1 = -5.0\mu$C and $q_2 = +2.0\mu$C are positioned at two corners of a rectangle as shown.



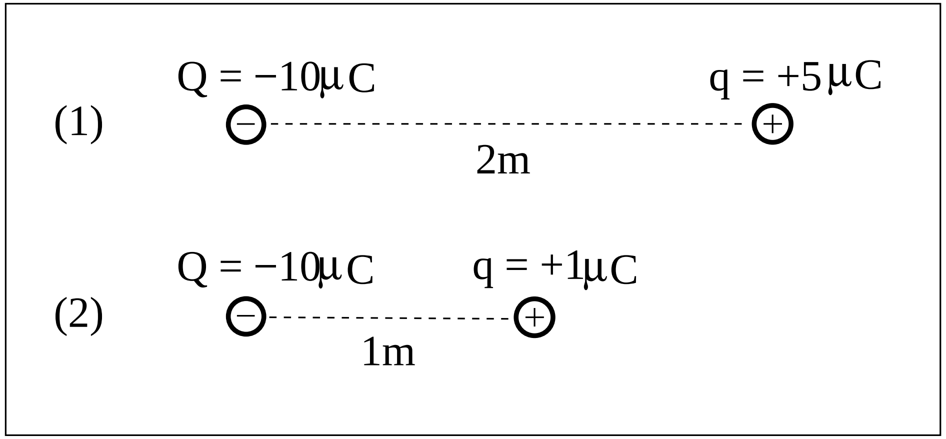
(a) Find the electric potential at the corners A and B.

(b) Find the electric field at point B.

(c) How much work is required to move a point charge $q_3 = +3\mu$C from B to A?

Electric Potential and Potential Energy: Application (4)

A positive point charge q is positioned in the electric field of a negative point charge Q.



(a) In which configuration is the charge q positioned in the stronger electric field?

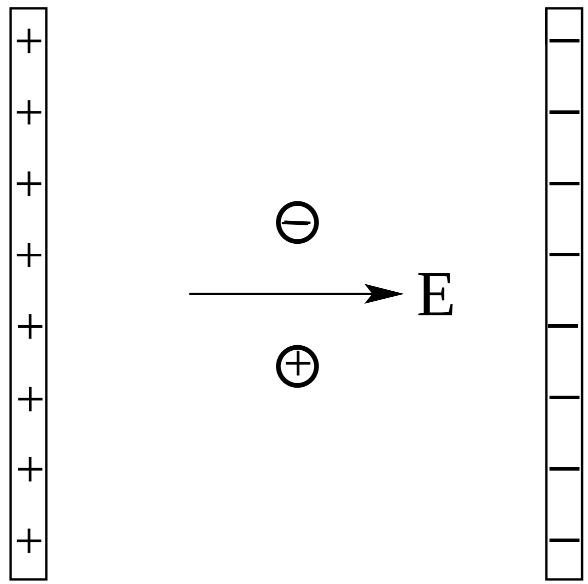
(b) In which configuration does the charge q experience the stronger force?

(c) In which configuration is the charge q positioned at the higher electric potential?

(d) In which configuration does the charge q have the higher potential energy?

Electric Potential and Potential Energy: Application (5)

An electron and a proton are released from rest midway between oppositely charged plates.



(a) Name the particle(s) which move(s) from high to low electric potential.

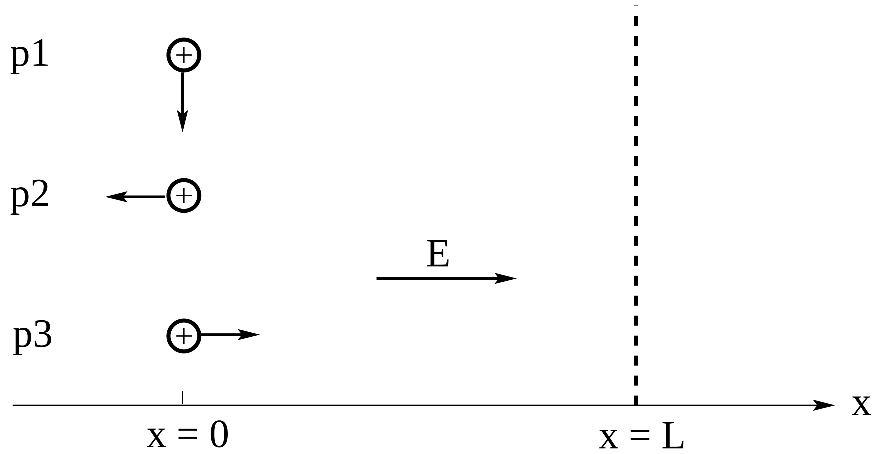
(b) Name the particle(s) whose electric potential energy decrease(s).

(c) Name the particle(s) which hit(s) the plate in the shortest time.

(d) Name the particle(s) which reach(es) the highest kinetic energy before impact.

Electric Potential and Potential Energy: Application (6)

Three protons are projected from $x = 0$ with equal initial speed v_0 in different directions. They all experience the force of a uniform horizontal electric field $\vec{E}$. Ultimately, they all hit the vertical screen at $x = L$.



(a) Which proton travels the longest time?

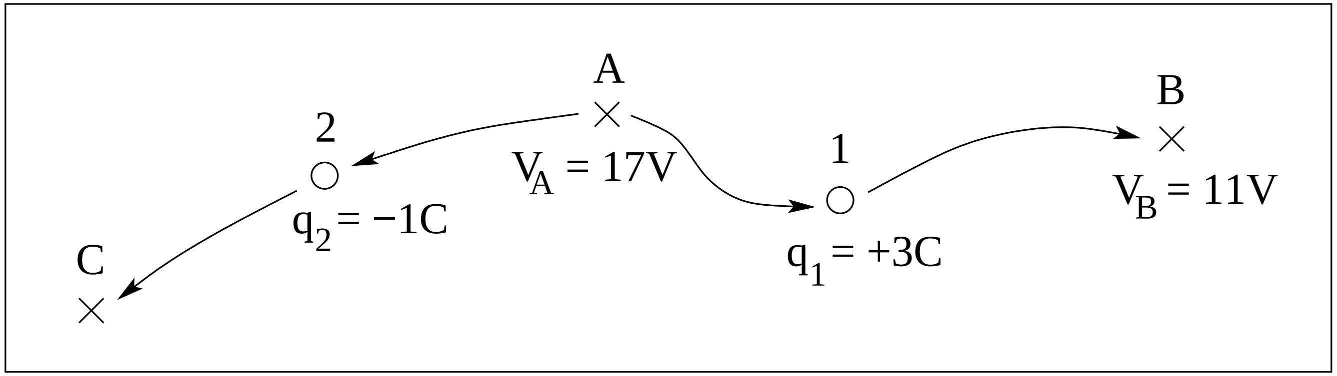
(b) Which proton travels the longest path?

(c) Which particle has the highest speed when it hits the screen?

Two of the questions are easy, one is hard.

Electric Potential and Potential Energy: Application (7)

Consider a region of nonuniform electric field. Charged particles 1 and 2 start moving from rest at point A in opposite directions along the paths shown.



From the information given in the figure...

(a) find the kinetic energy K_1 of particle 1 when it arrives at point B,

(b) find the electric potential V_C at point C if we know that particle 2 arrives there with kinetic energy $K_2 = 8J$.

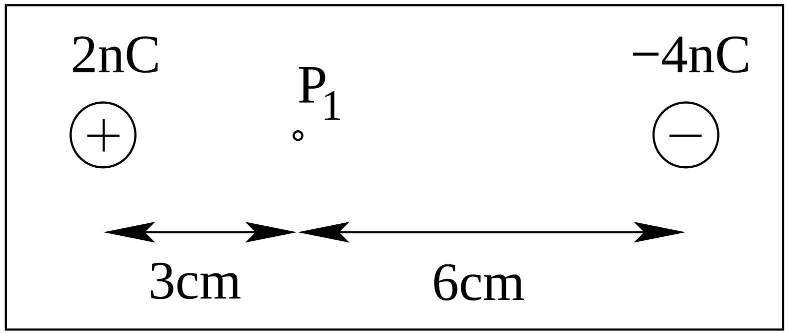
Electric Potential and Potential Energy: Application (8)

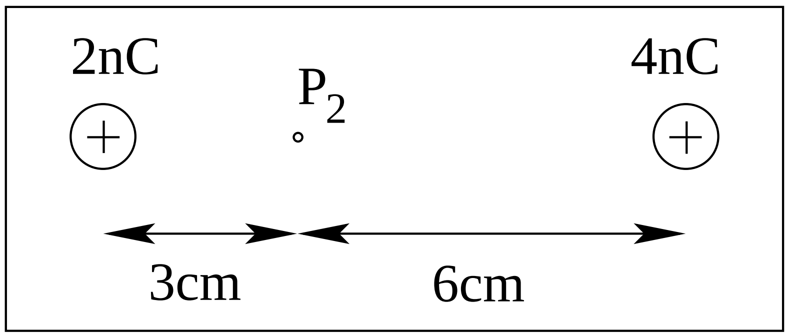
(a) Is the electric potential at points P_1, P_2 **positive** or **negative** or **zero**?

(b) Is the potential energy of a negatively charged particle at points P_1, P_2 **positive** or **negative** or **zero**?

(c) Is the electric field at points P_1, P_2 directed **left** or **right** or is it **zero**?

(d) Is the force on a negatively charged particle at points P_1 and P_2 directed **left** or **right** or is it **zero**?





Electric Potential and Potential Energy: Application (9)

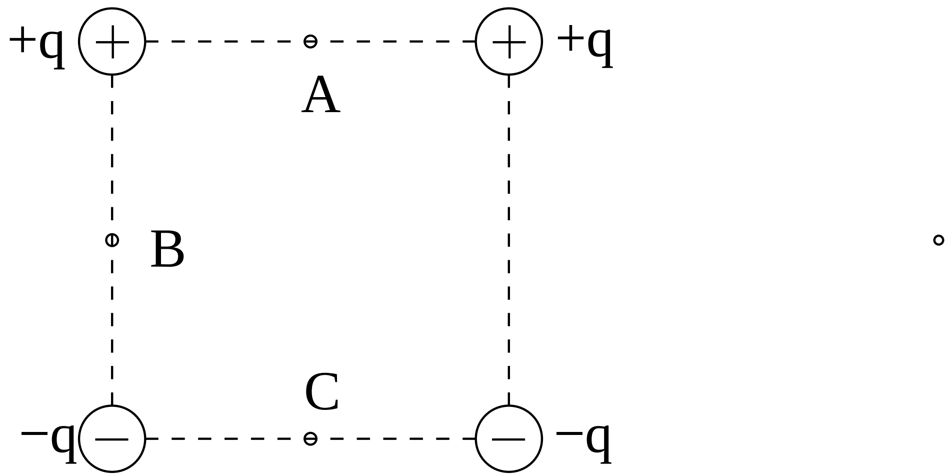
Consider four point charges of equal magnitude positioned at the corners of a square as shown. Answer the following questions for points A, B, C.

(1) Which point is at the highest electric potential?

(2) Which point is at the lowest electric potential?

(3) At which point is the electric field the strongest?

(4) At which point is the electric field the weakest?



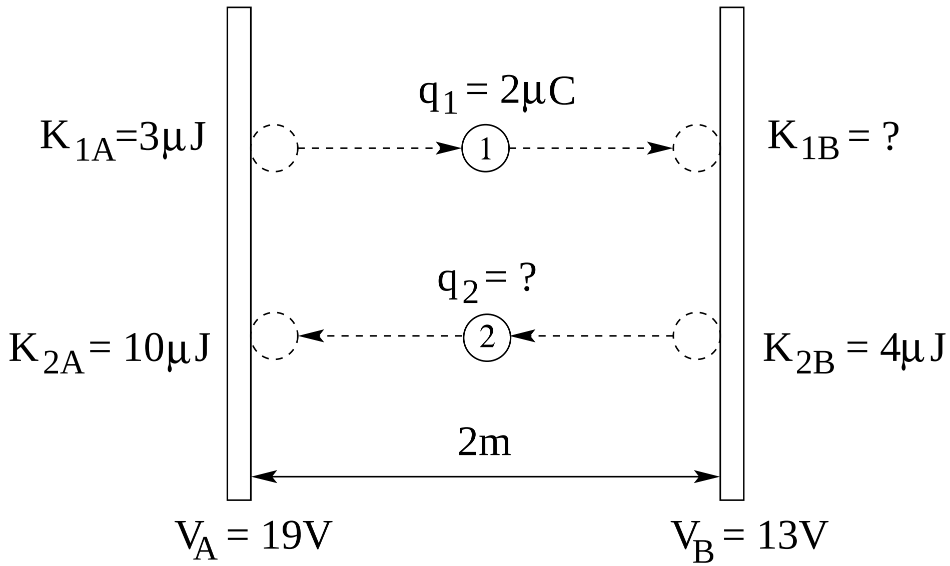
The charged particles 1 and 2 move between the charged conducting plates A and B in opposite directions.

From the information given in the figure...

(a) find the kinetic energy K_{1B} of particle 1,

(b) find the charge q_2 of particle 2,

(c) find the direction and magnitude of the electric field $\vec{E}$ between the plates.

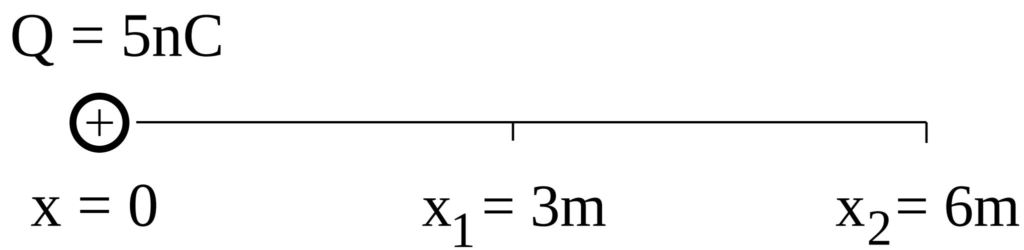


Intermediate Exam I: Problem #2 (Spring ’05)

Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3\text{m}$
and the electric potiential V_2 at position $x_2 = 6\text{m}$.

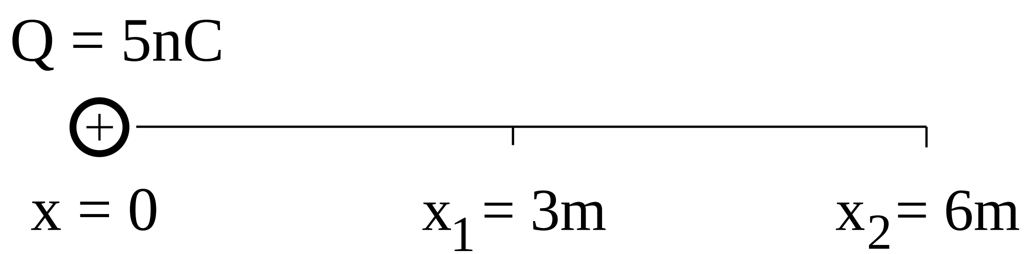
(b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3\text{m}$
and the electric potiential V_2 at position $x_2 = 6\text{m}$.

(b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



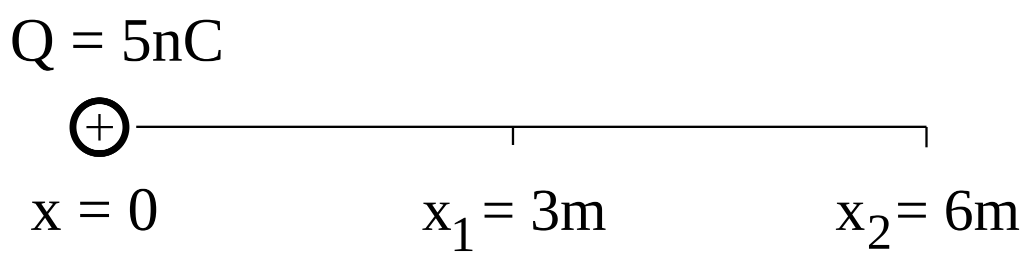
Solution:

(a) $V_1 = k\dfrac{Q}{x_1} = 15\text{V}, \quad V_2 = k\dfrac{Q}{x_2} = 7.5\text{V}.$

Consider a point charge $Q = 5$nC fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3$m
and the electric potiential V_2 at position $x_2 = 6$m.

(b) If a charged particle ($q = 4$nC, $m = 1.5$ng) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



Solution:

(a) $V_1 = k\dfrac{Q}{x_1} = 15\text{V}, \quad V_2 = k\dfrac{Q}{x_2} = 7.5\text{V}.$

(b) $\Delta U = q(V_2 - V_1) = (4\text{nC})(-7.5\text{V}) = -30\text{nJ} \quad \Rightarrow \Delta K = -\Delta U = 30\text{nJ}.$

$$\Delta K = K_2 = \frac{1}{2}mv_2^2 \quad \Rightarrow v_2 = \sqrt{\frac{2K_2}{m}} = 200\text{m/s}.$$

Unit Exam I: Problem #1 (Fall '10)

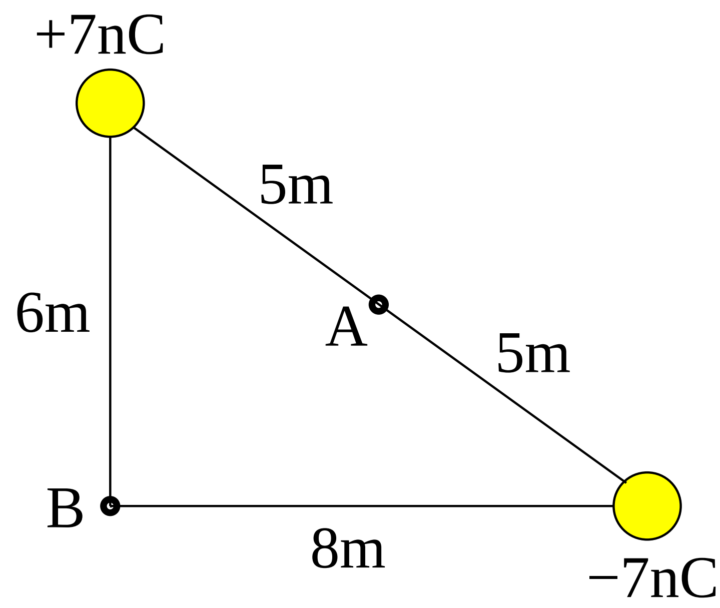
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

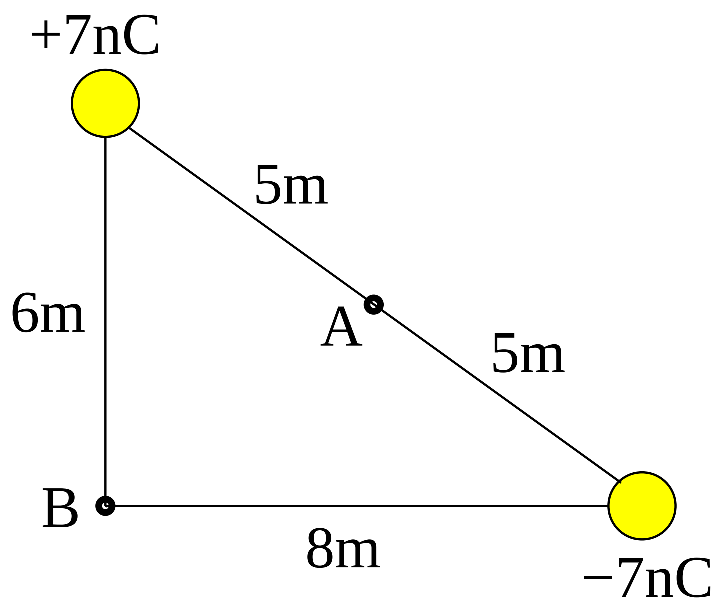
(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.



Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.
(b) Find the electric potential at point A.
(c) Find the magnitude of the electric field at point B.
(d) Find the electric potential at point B.



Solution:

(a) $E_A = 2k\dfrac{|7\mathrm{nC}|}{(5\mathrm{m})^2} = 2(2.52\mathrm{V/m}) = 5.04\mathrm{V/m}.$

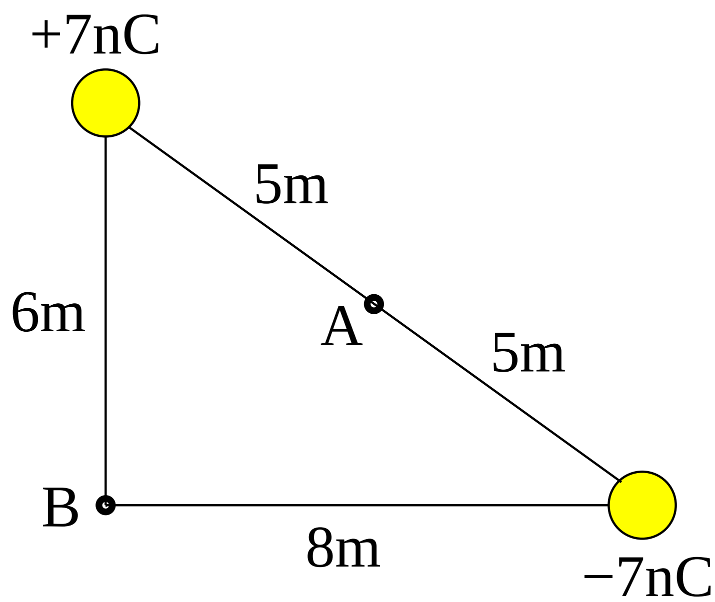
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.



Solution:

(a) $E_A = 2k\dfrac{|7\mathrm{nC}|}{(5\mathrm{m})^2} = 2(2.52\mathrm{V/m}) = 5.04\mathrm{V/m}.$

(b) $V_A = k\dfrac{(+7\mathrm{nC})}{5\mathrm{m}} + k\dfrac{(-7\mathrm{nC})}{5\mathrm{m}} = 12.6\mathrm{V} - 12.6\mathrm{V} = 0.$

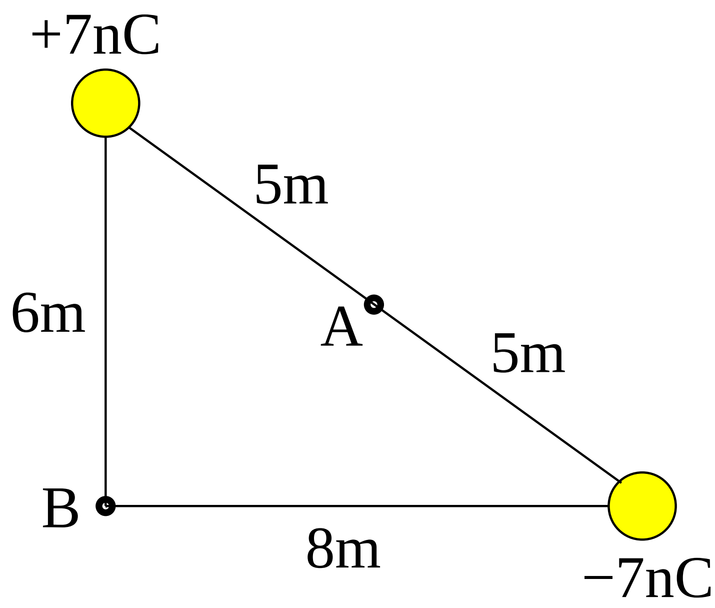
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.



Solution:

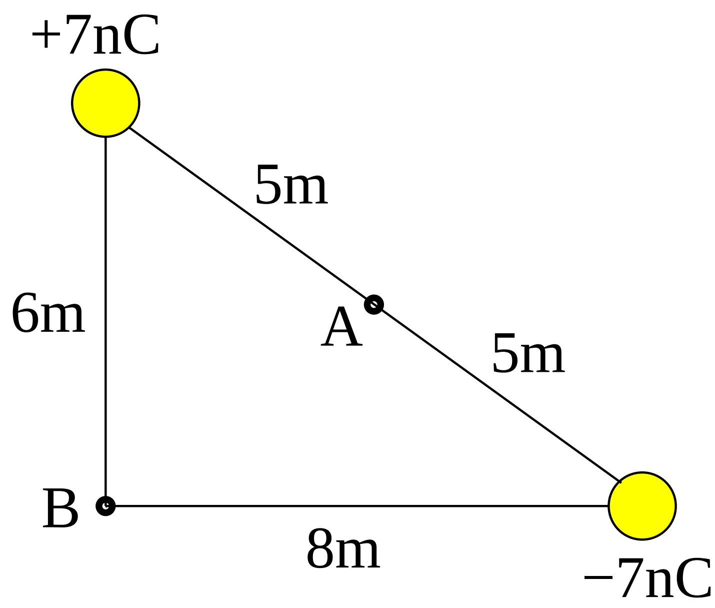
(a) $E_A = 2k\dfrac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$

(b) $V_A = k\dfrac{(+7\text{nC})}{5\text{m}} + k\dfrac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$

(c) $E_B = \sqrt{\left(k\dfrac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k\dfrac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$

Unit Exam I: Problem #1 (Fall ’10)

Consider two point charges positioned as shown.



(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.

Solution:

(a) $E_A = 2k\dfrac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$

(b) $V_A = k\dfrac{(+7\text{nC})}{5\text{m}} + k\dfrac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$

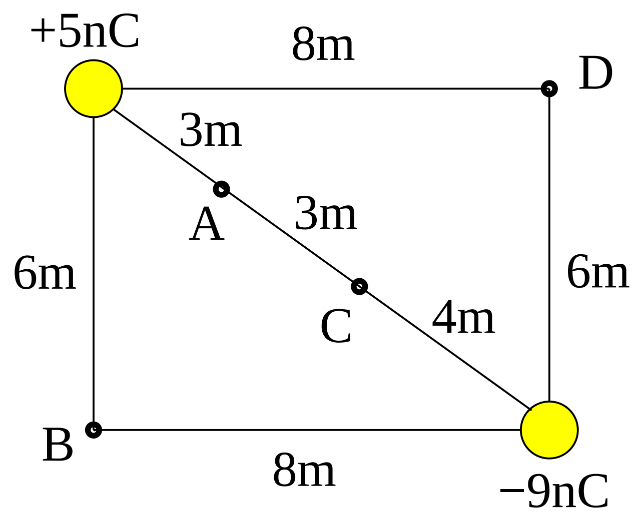
(c) $E_B = \sqrt{\left(k\dfrac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k\dfrac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$

(d) $V_B = k\dfrac{(+7\text{nC})}{6\text{m}} + k\dfrac{(-7\text{nC})}{8\text{m}} = 10.5\text{V} - 7.9\text{V} = 2.6\text{V}.$

Unit Exam I: Problem #1 (Spring '14)

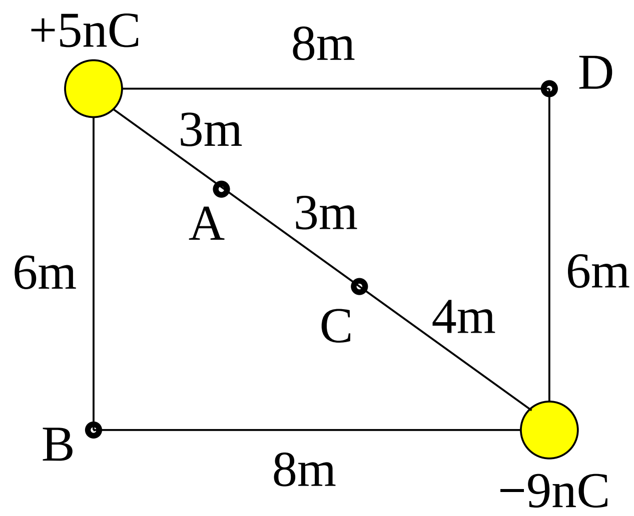
Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.



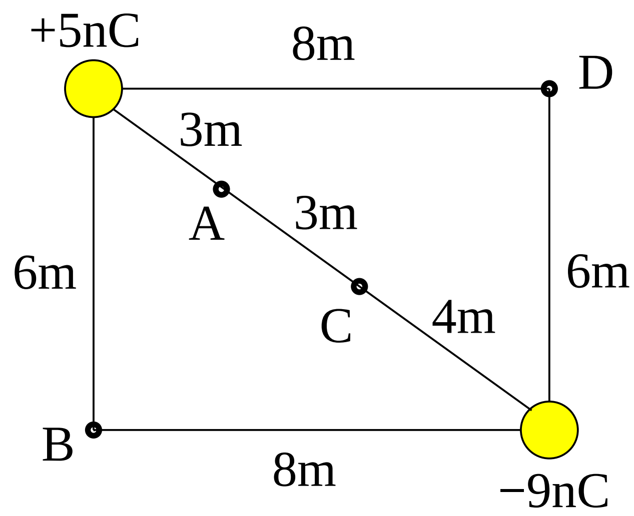
Solution:

- $E_A = k\dfrac{|5\text{nC}|}{(3\text{m})^2} + k\dfrac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$

Unit Exam I: Problem #1 (Spring '14)

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.

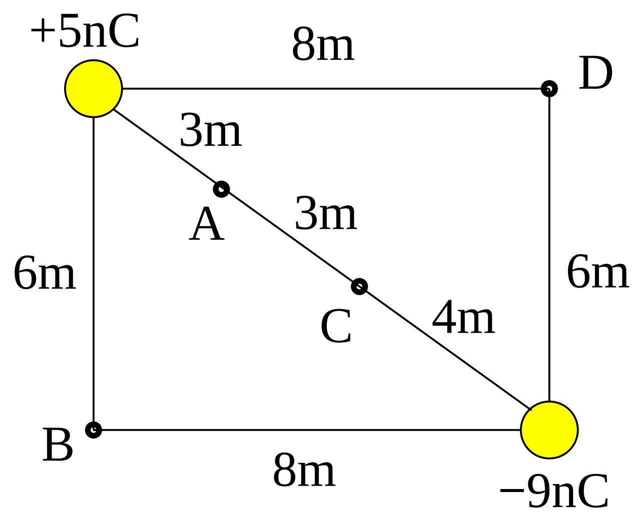


Solution:

- $E_A = k\frac{|5\mathrm{nC}|}{(3\mathrm{m})^2} + k\frac{|-9\mathrm{nC}|}{(7\mathrm{m})^2} = 5.00\mathrm{V/m} + 1.65\mathrm{V/m} = 6.65\mathrm{V/m}.$
- $V_B = k\frac{(+5\mathrm{nC})}{6\mathrm{m}} + k\frac{(-9\mathrm{nC})}{8\mathrm{m}} = 7.50\mathrm{V} - 10.13\mathrm{V} = -2.63\mathrm{V}.$

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.

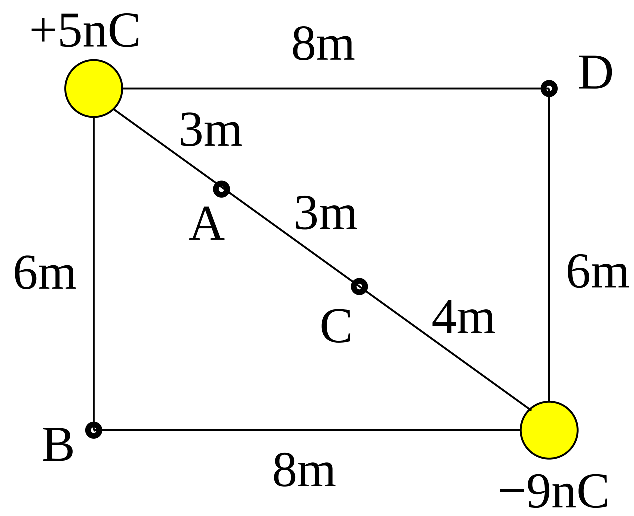


Solution:

- $E_A = k\dfrac{|5\text{nC}|}{(3\text{m})^2} + k\dfrac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k\dfrac{(+5\text{nC})}{6\text{m}} + k\dfrac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$
- $E_C = k\dfrac{|5\text{nC}|}{(6\text{m})^2} + k\dfrac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}.$

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.



Solution:

- $E_A = k\dfrac{|5\text{nC}|}{(3\text{m})^2} + k\dfrac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k\dfrac{(+5\text{nC})}{6\text{m}} + k\dfrac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$
- $E_C = k\dfrac{|5\text{nC}|}{(6\text{m})^2} + k\dfrac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}.$
- $V_D = k\dfrac{(+5\text{nC})}{8\text{m}} + k\dfrac{(-9\text{nC})}{6\text{m}} = 5.63\text{V} - 13.5\text{V} = -7.87\text{V}.$