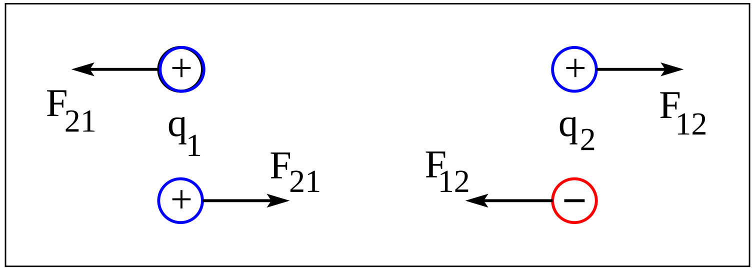
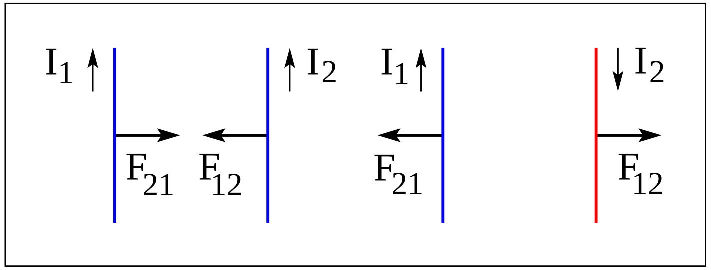
Electricity

- Electric charges generate an electric field.
- The electric field exerts a force on other electric charges.



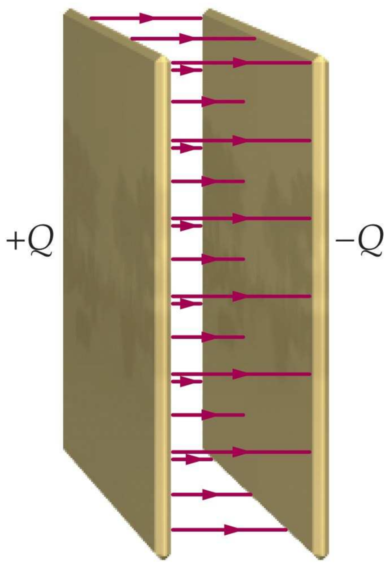
Magnetism

- Electric currents generate a magnetic field.
- The magnetic field exerts force on other electric currents.



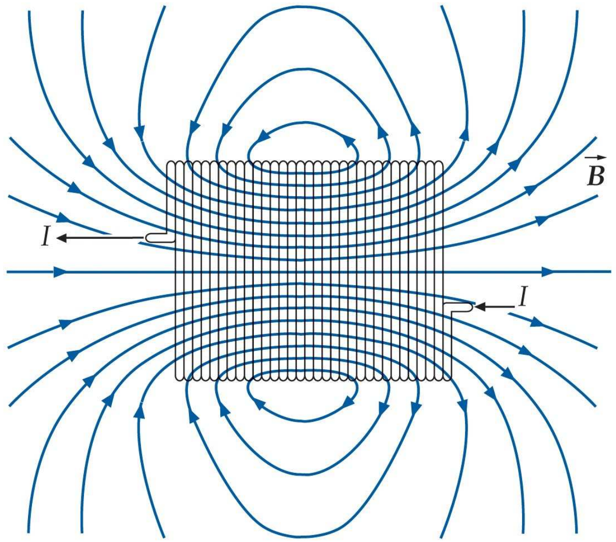
Capacitor

The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.



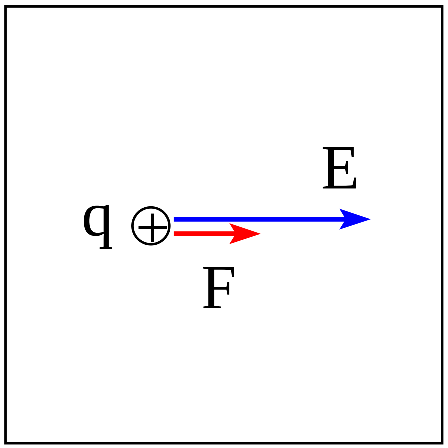
Solenoid

The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.



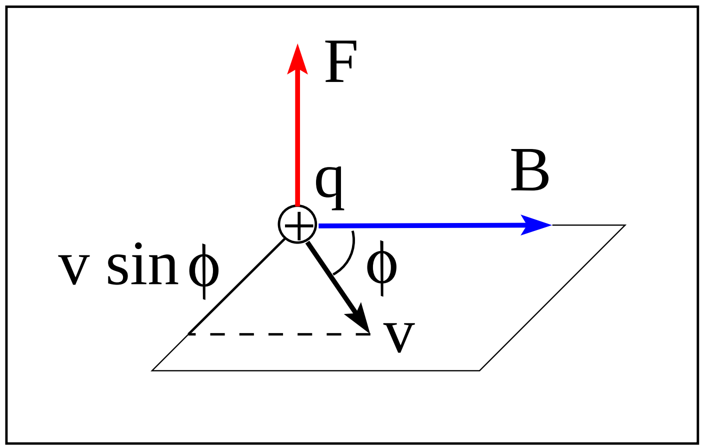
Electric Force

- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of E: 1N/C=1V/m



Magnetic Force

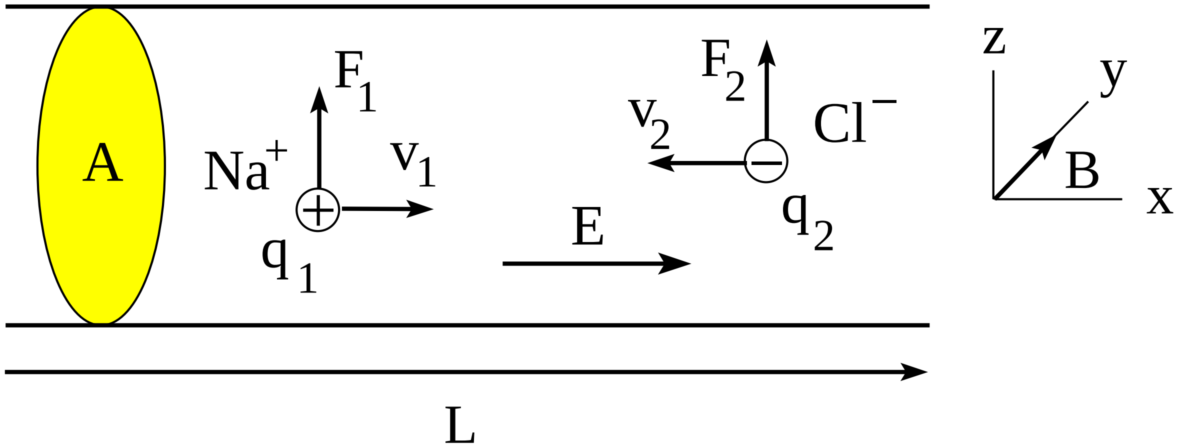
- $\vec{F} = q\vec{v} \times \vec{B}, \quad F = qvB\sin\phi$
- magnetic force is perpendicular to magnetic field
- SI unit of B: 1Ns/Cm=1T (Tesla)
- 1T=10^4G (Gauss)



Magnetic Force on Current-Carrying Conductor

Consider drift of Na^+ and Cl^- ions in a plastic pipe filled with salt water.

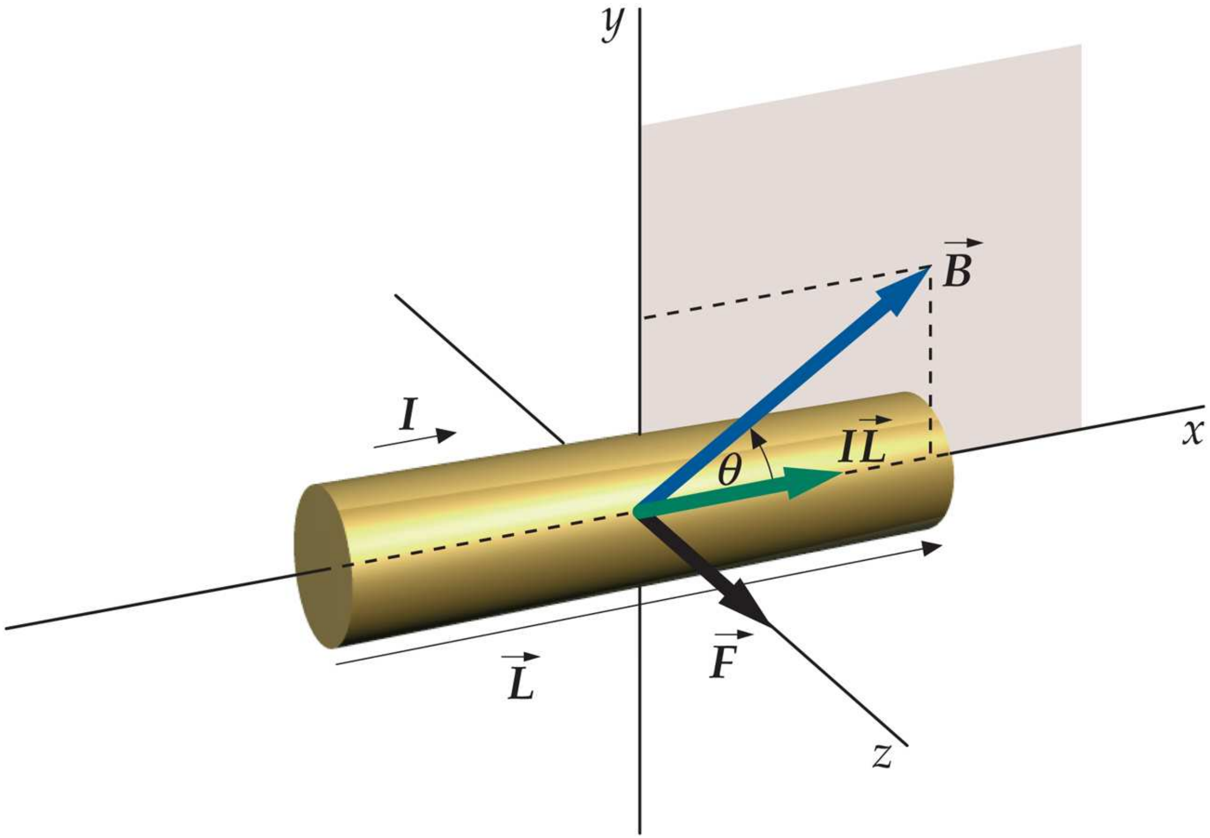
- $v_{1x} > 0,\ v_{2x} < 0$: drift velocities; $q_1 > 0,\ q_2 < 0$: charge on ions
- $n_1,\ n_2$: number of charge carriers per unit volume



- Electric current through A: $I = A(n_1 q_1 v_{1x} + n_2 q_2 v_{2x})$
- Force on Na^+: $\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B} \ \Rightarrow\ F_{1z} = q_1 v_{1x} B_y$
- Force on Cl^-: $\vec{F}_2 = q_2 \vec{v}_2 \times \vec{B} \ \Rightarrow\ F_{2z} = q_2 v_{2x} B_y$
- Force on current-carrying pipe: $F_z = (n_1 q_1 v_{1x} + n_2 q_2 v_{2x}) A L B_y = I L B_y$
- Vector relation: $\vec{F} = I \vec{L} \times \vec{B}$

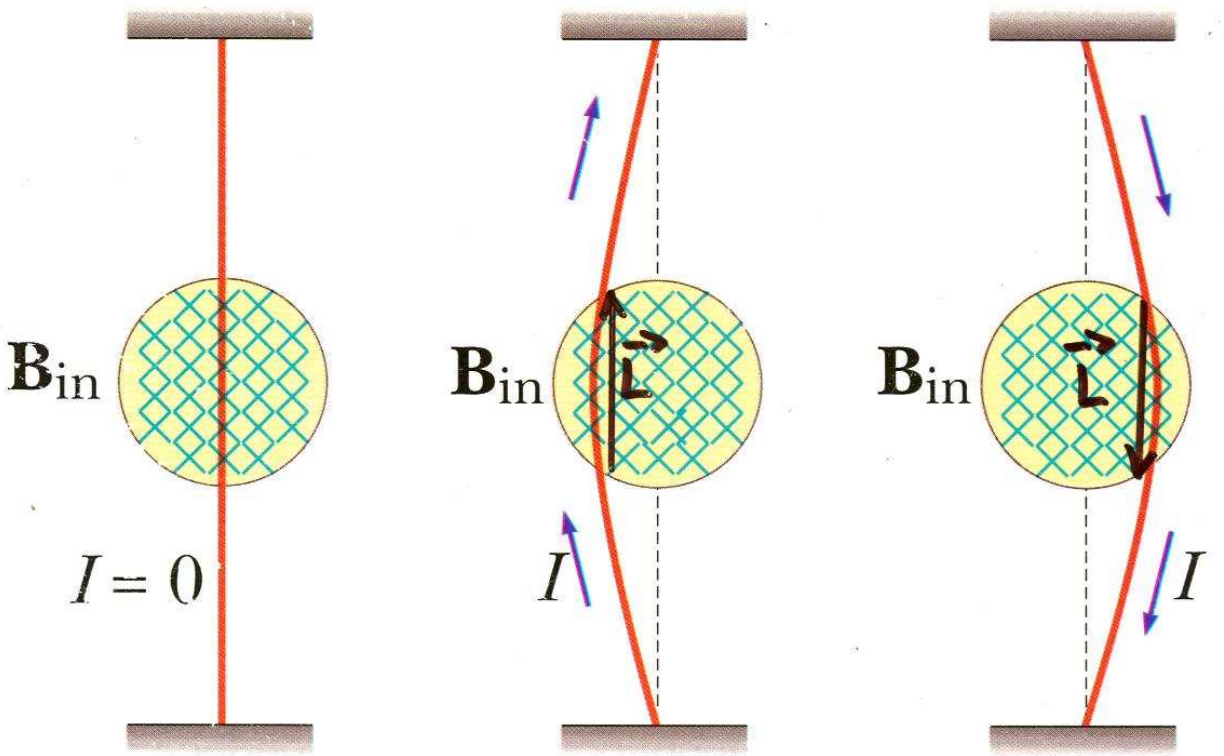
Direction of Magnetic Force

$$\vec{F} = I\vec{L} \times \vec{B}$$



Direction of Magnetic Force

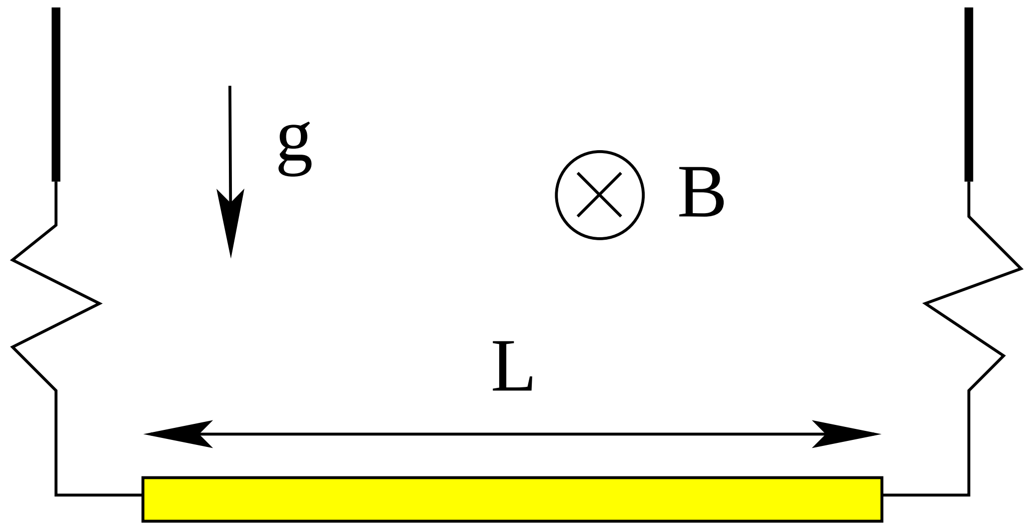
$$\vec{F} = I\vec{L} \times \vec{B}$$



Magnetic Force Application (1)

A wire of length $L = 62$cm and mass $m = 13$g is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440$T pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?



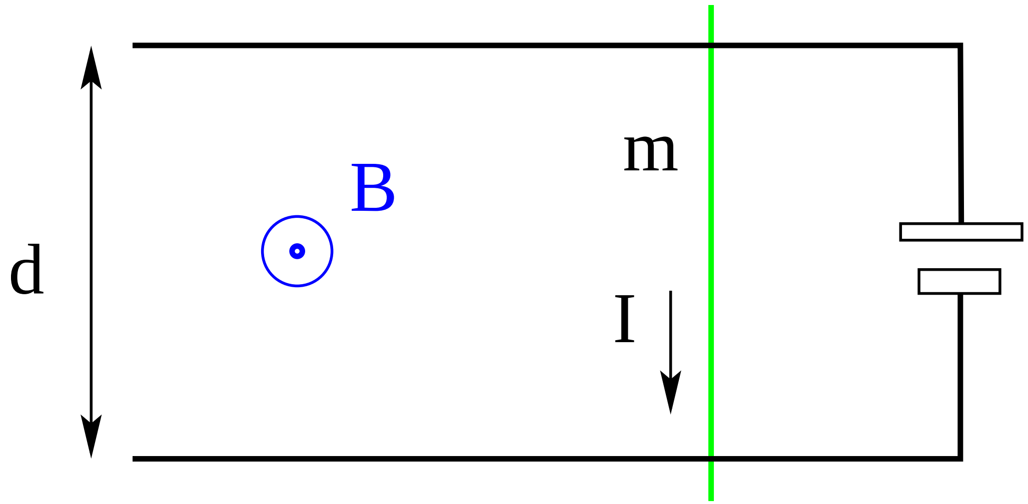
Magnetic Force Application (2)

A metal wire of mass $m = 1.5$kg slides without friction on two horizontal rails spaced a distance $d = 3$m apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24$mT pointing out of the plane.

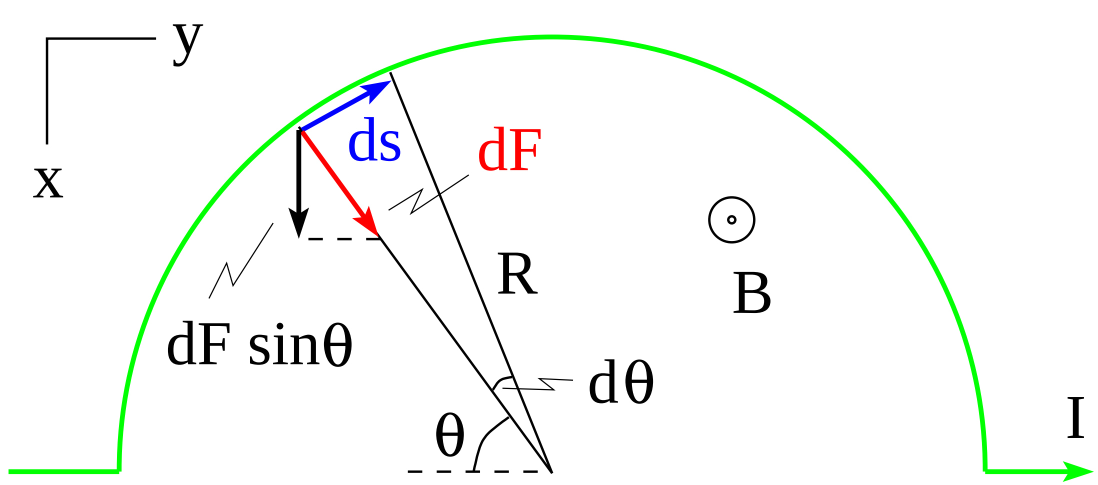
A constant current $I = 12$A flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

- Find the direction and magnitude of the velocity of the wire at time $t = 5$s.



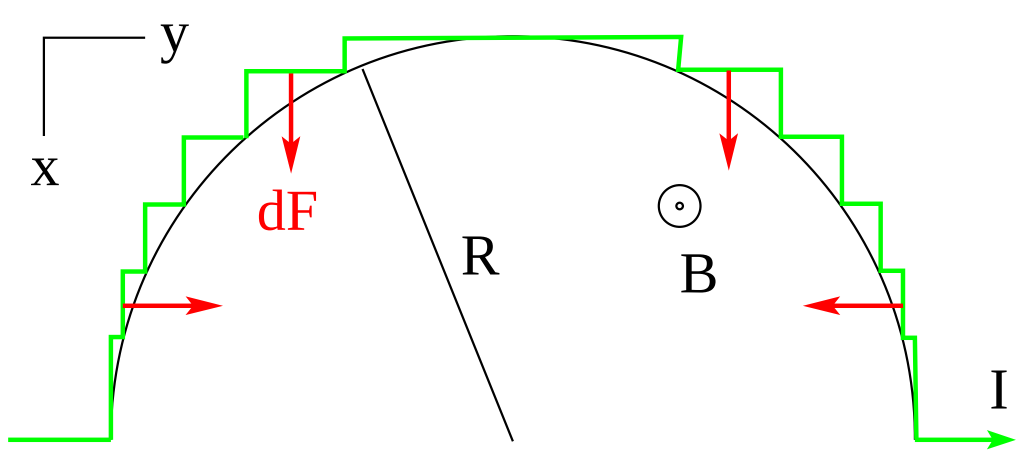
Fancy solution:

- Uniform magnetic field $\vec{B}$ points out of the plane.
- Magnetic force on segment ds: $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF \sin\theta$ and $dF_y = dF \cos\theta$ along semicircle.
- $F_x = IBR \int_0^\pi \sin\theta d\theta = 2IBR, \quad F_y = IBR \int_0^\pi \cos\theta d\theta = 0.$



Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$. The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.



Magnetic Force Application (5)

Inside the cube there is a magnetic field $\vec{B}$ directed vertically up.

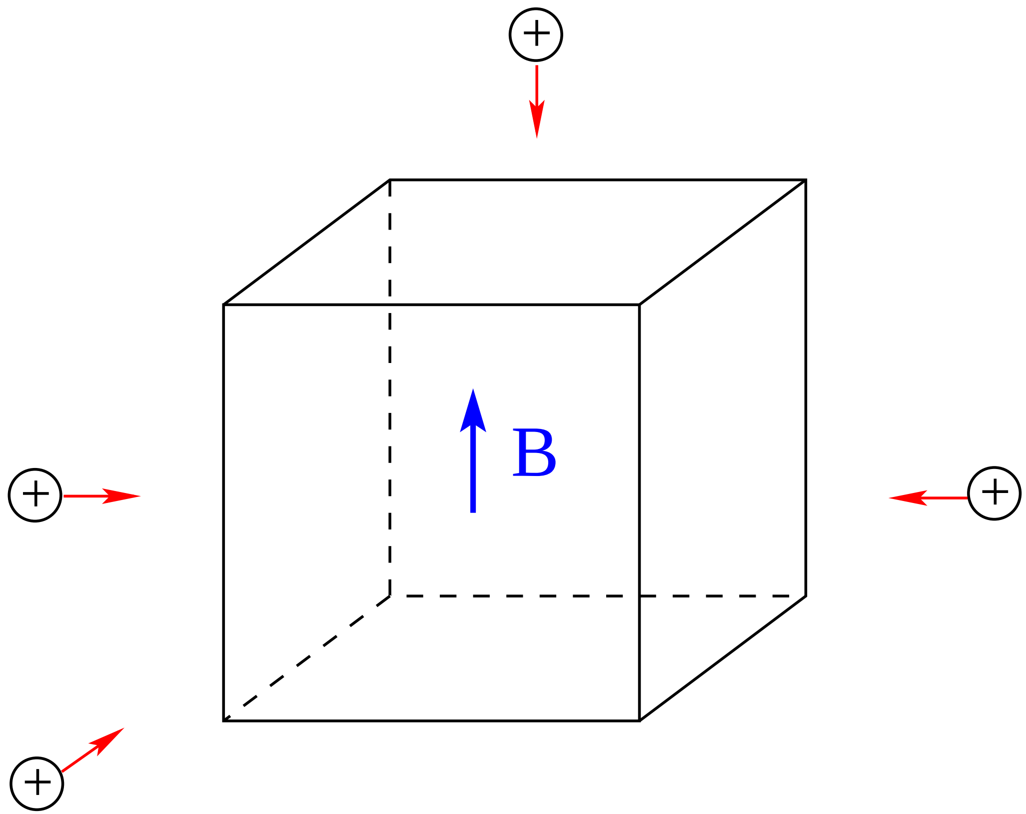
Find the direction of the magnetic force experienced by a proton entering the cube

(a) from the left,

(b) from the front,

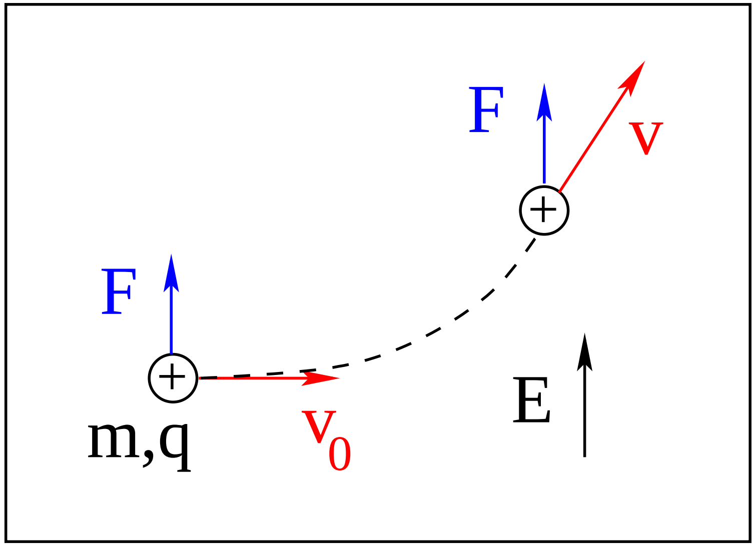
(c) from the right,

(d) from the top.



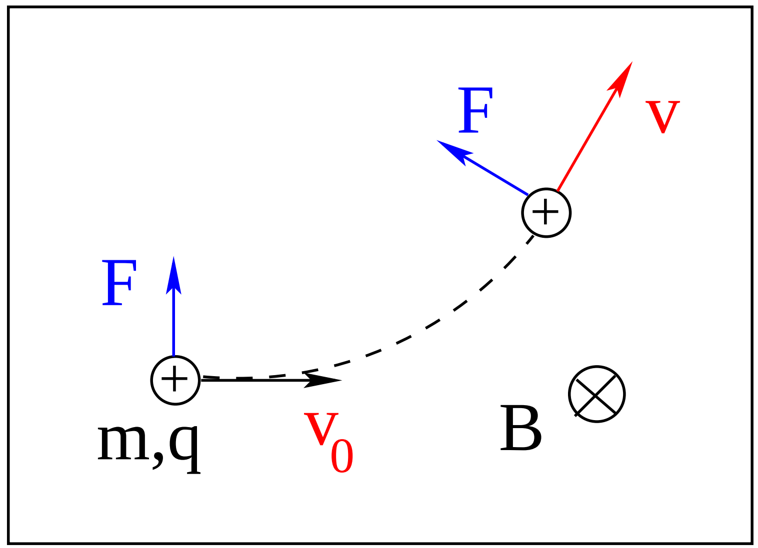
Charged Particle Moving in Uniform Electric Field

- Electric field $\vec{E}$ is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{E} =$ const.
- Horizontal motion: $a_x = 0 \;\Rightarrow\; v_x(t) = v_0 \;\Rightarrow\; x(t) = v_0 t$
- Vertical motion: $a_y = \frac{q}{m}E \;\Rightarrow\; v_y(t) = a_y t \;\Rightarrow\; y(t) = \frac{1}{2}a_y t^2$
- The path is parabolic: $y = \left(\frac{qE}{2mv_0^2}\right)x^2$
- $\vec{F}$ changes direction and magnitude of $\vec{v}$.



Charged Particle Moving in Uniform Magnetic Field

- Magnetic field $\vec{B}$ is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \;\Rightarrow\; \vec{F}$ changes direction of $\vec{v}$ only $\;\Rightarrow\; v = v_0$.
- $\vec{F}$ is the centripetal force of motion along circular path.
- Radius: $\dfrac{mv^2}{r} = qvB \;\Rightarrow\; r = \dfrac{mv}{qB}$
- Angular velocity: $\omega = \dfrac{v}{r} = \dfrac{qB}{m}$
- Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB}$



Charged Particle in Circular Motion

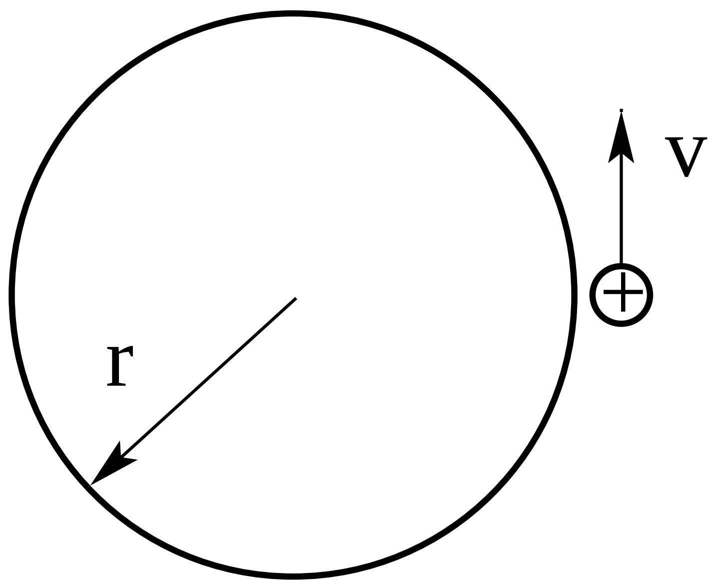
A proton with speed $v = 3.00 \times 10^5$m/s orbits just outside a charged conducting sphere of radius $r = 1.00$cm.

(a) Find the force F acting on the proton.

(b) Find the charge per unit area σ on the surface of the sphere.

(c) Find the total charge Q on the sphere.

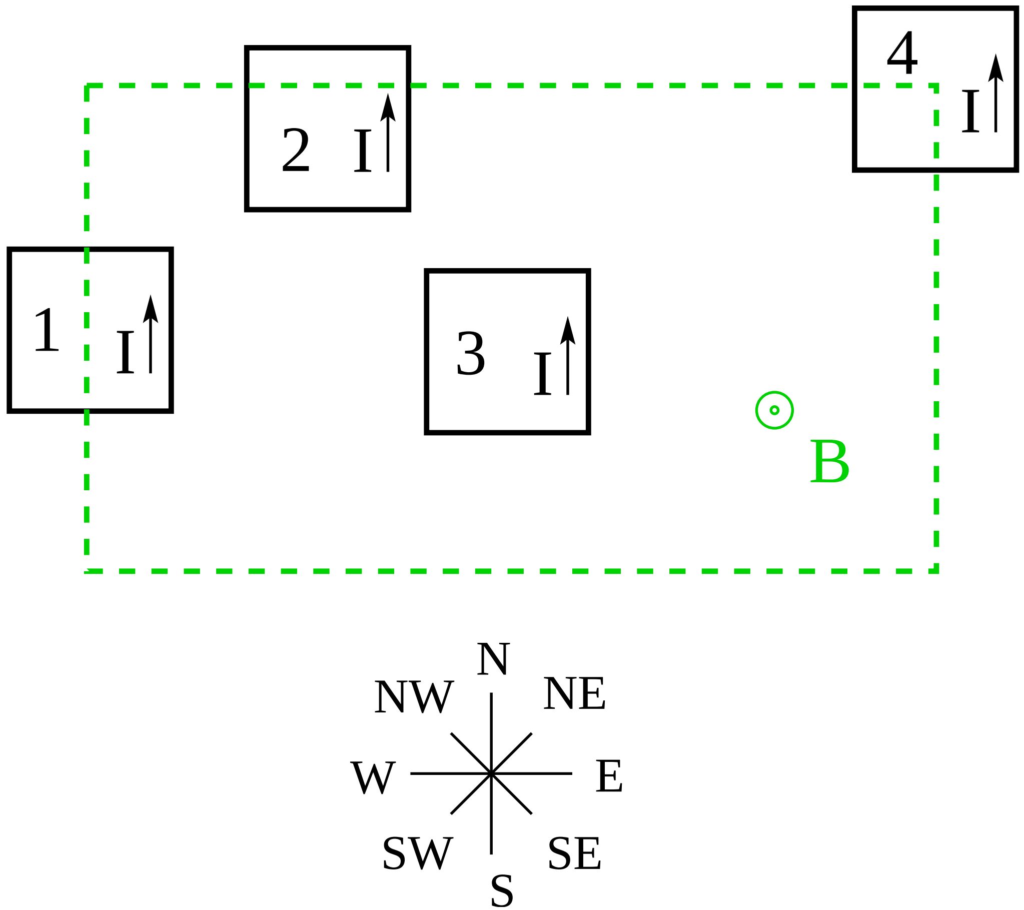
Note: Charged particles in circular motion lose energy through radiation. This effect is ignored here.



Magnetic Force Application (3)

The dashed rectangle marks a region of uniform magnetic field $\vec{B}$ pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current I.



Velocity Selector

A charged particle is moving horizontally into a region with "crossed" uniform fields:

- an electric field $\vec{E}$ pointing down,
- a magnetic field $\vec{B}$ pointing into the plane.

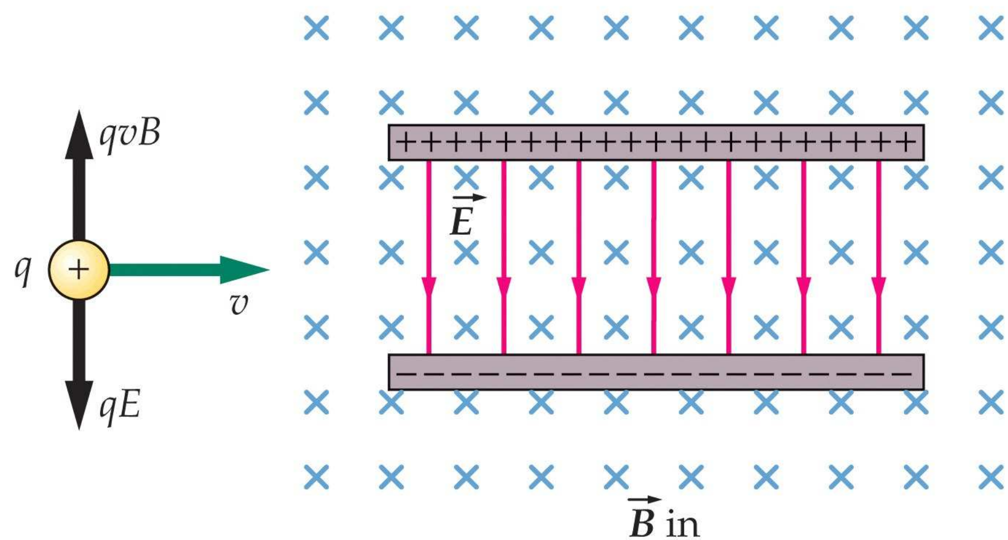
Forces experienced by particle:

- electric force $F = qE$ pointing down,
- magnetic force $B = qvB$ pointing up.

Forces in balance: $qE = qvB$.

Selected velocity: $v = \dfrac{E}{B}$.

Trajectories of particles with selected velocity are not bent.



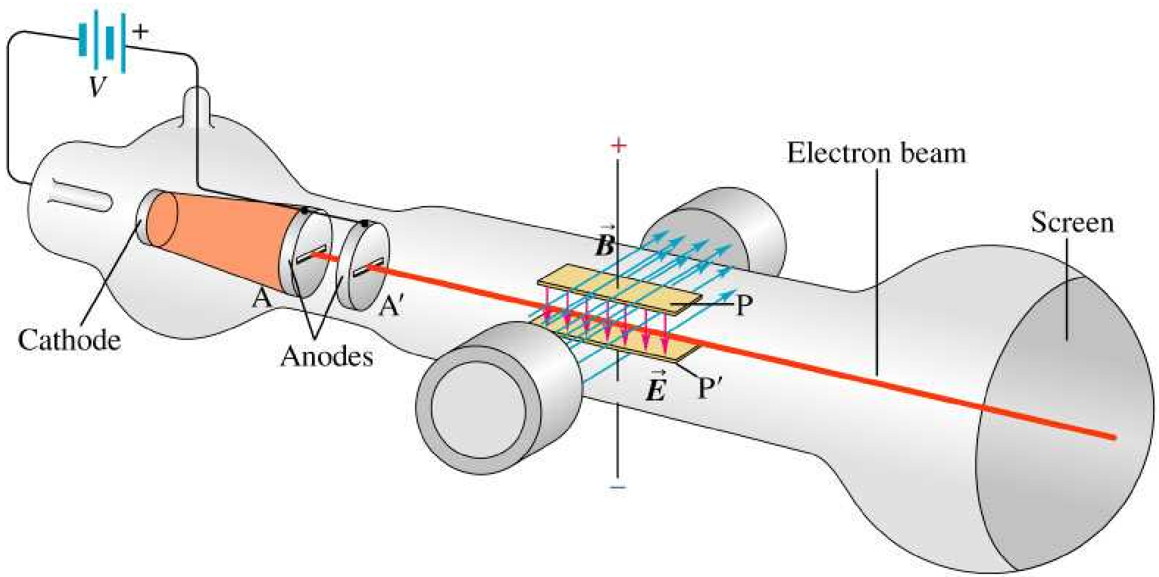
Measurement of e/m for Electron

First experiment by J. J. Thomson (1897)
Method used here: velocity selector

Equilibrium of forces: $eE = evB \quad \Rightarrow \; v = \dfrac{E}{B}$

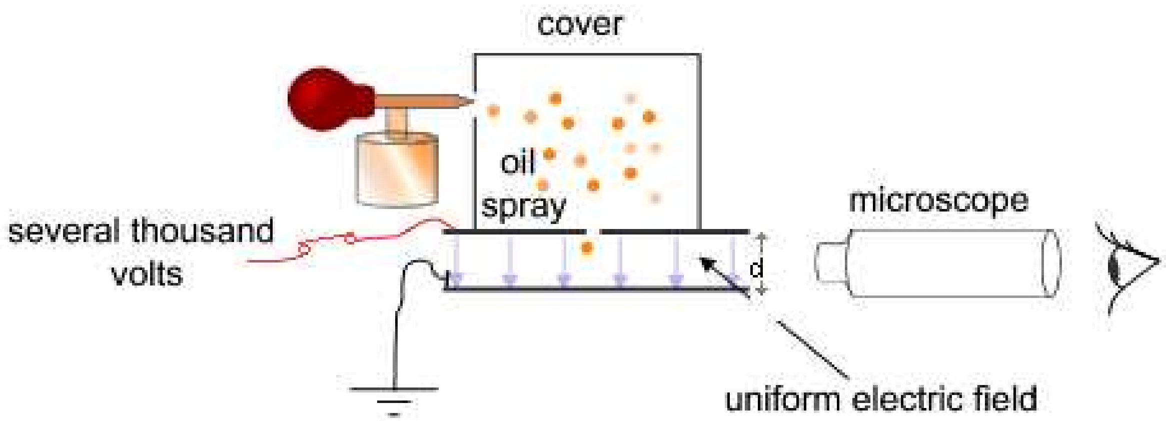
Work-energy relation: $eV = \dfrac{1}{2}mv^2 \quad \Rightarrow \; v = \sqrt{\dfrac{2eV}{m}}$

Eliminate v: $\dfrac{e}{m} = \dfrac{E^2}{2VB^2} \simeq 1.76 \times 10^{11}\mathrm{C/kg}$



Measurement of e and m for Electron

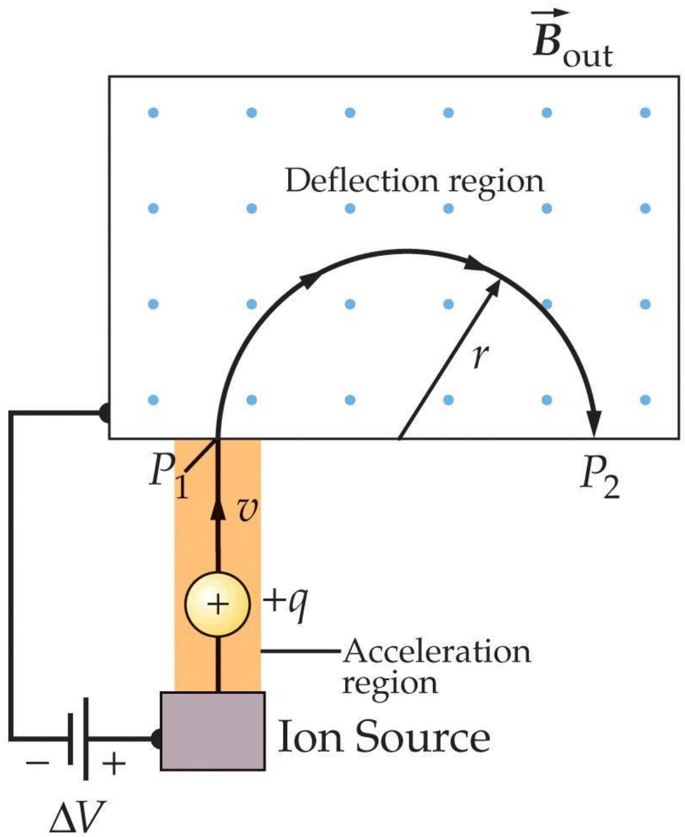
First experiment by R. Millikan (1913)
Method used here: balancing weight and electric force on oil drop
Radius of oil drop: $r = 1.64\mu\mathrm{m}$
Mass density of oil: $\rho = 0.851\mathrm{g/cm}^3$
Electric field: $E = 1.92 \times 10^5\mathrm{N/C}$
Mass of oil drop: $m = \frac{4\pi}{3}r^3\rho = 1.57 \times 10^{-14}\mathrm{kg}$
Equilibrium of forces: $neE = mg$
Number of excess elementary charges (integer): $n = 5$
Elementary charge: $e = \frac{mg}{nE} \simeq 1.6 \times 10^{-19}\mathrm{C}$
Mass of electron: $m \simeq 9.1 \times 10^{-31}\mathrm{kg}$



Mass Spectrometer

Purpose: measuring masses of ions.

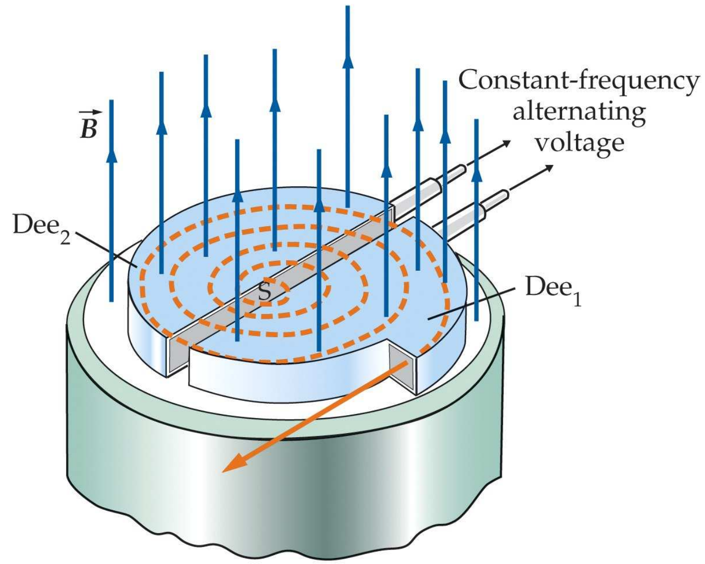
- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius r by magnetic field $\vec{B}$.
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$.



Cyclotron

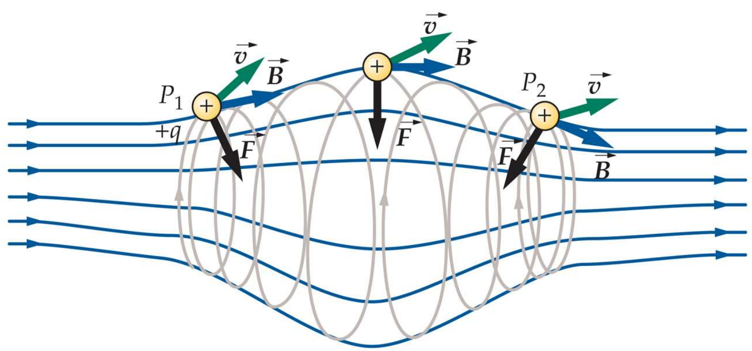
Purpose: accelerate charged particles to high energy.

- Low-energy protons are injected at S.
- Path is bent by magnetic field $\vec{B}$.
- Proton is energized by alternating voltage ΔV between Dee_1 and Dee_2.
- Proton picks up energy $\Delta K = e\Delta V$ during each half cycle.
- Path spirals out as velocity of particle increases: Radial distance is proportional to velocity: $r = \dfrac{mv}{eB}$.
- Duration of cycle stays is independent of r or v: cyclotron period: $T = \dfrac{2\pi m}{eB}$.
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of $\vec{B}$-field region.

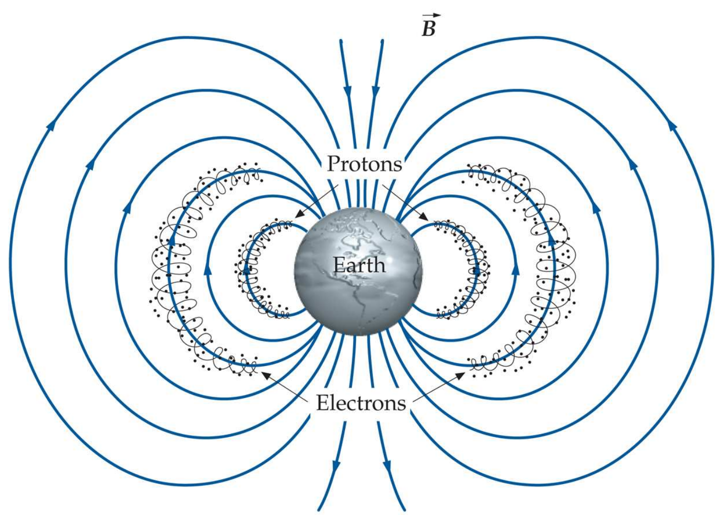


Magnetic Bottles

Moving charged particle
confined by
inhomogeneous magnetic field.

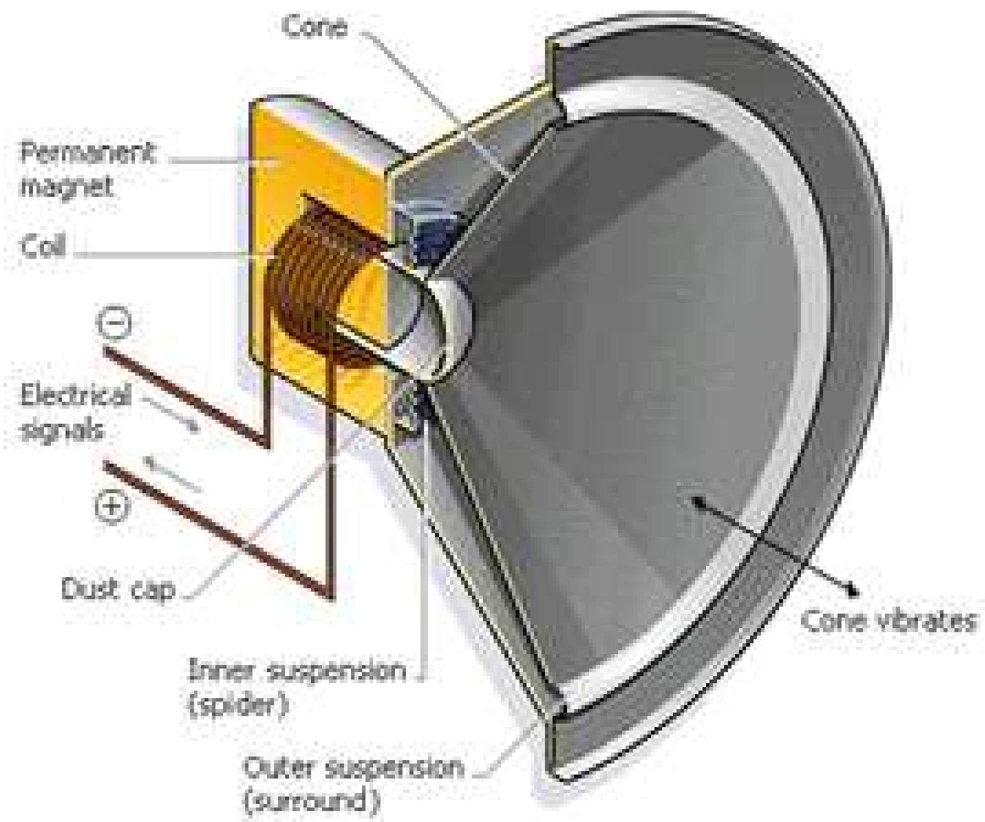


Van Allen belt:
trapped protons and electrons
in Earth's magnetic field.



Loudspeaker

Conversion of electric signal into mechanical vibration.

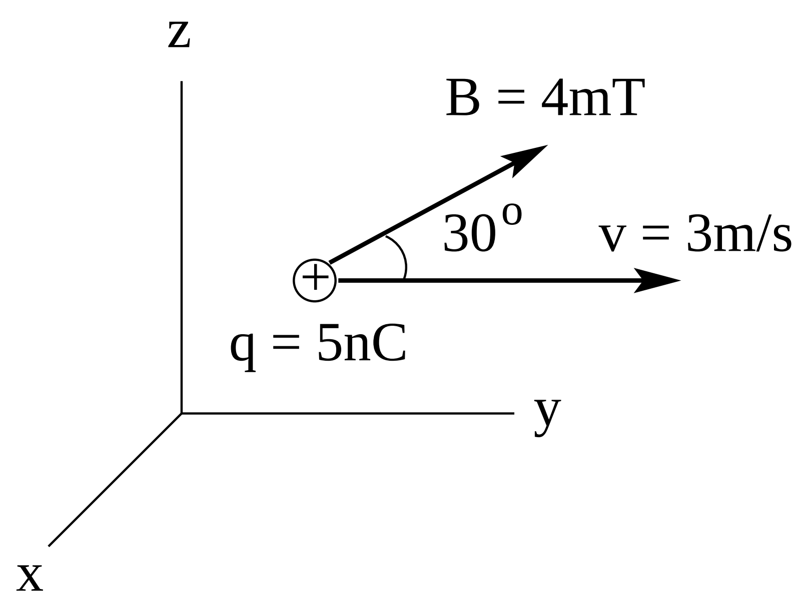


Intermediate Exam II: Problem #4 (Spring '05)

Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at $30°$ from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.

(b) Find the magnitude of the magnetic force acting on the particle.



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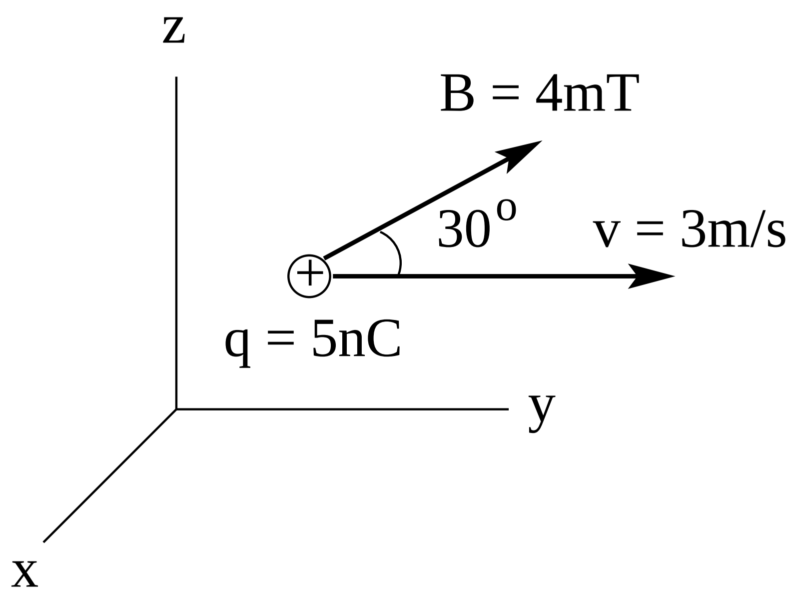

Solution:

(a) Use the right-hand rule: positive x-direction (front, out of page).

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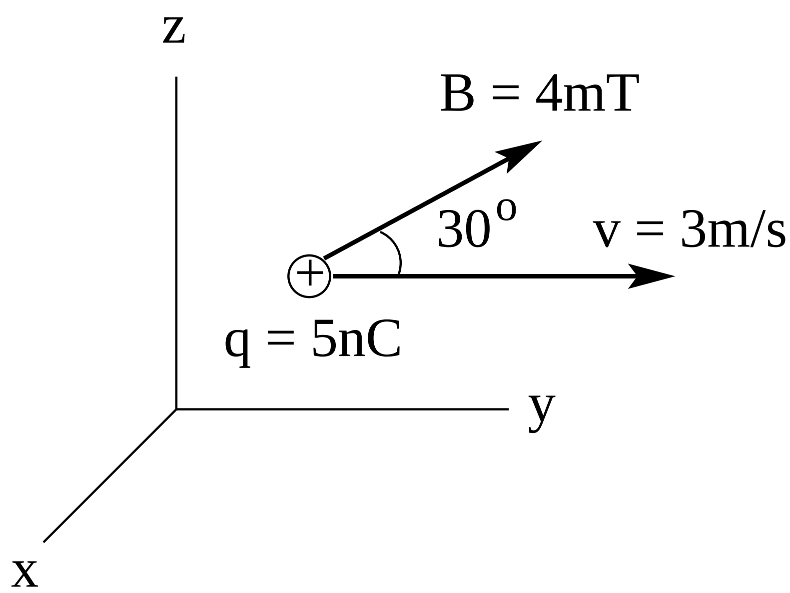


Solution:

(a) Use the right-hand rule: positive x-direction (front, out of page).

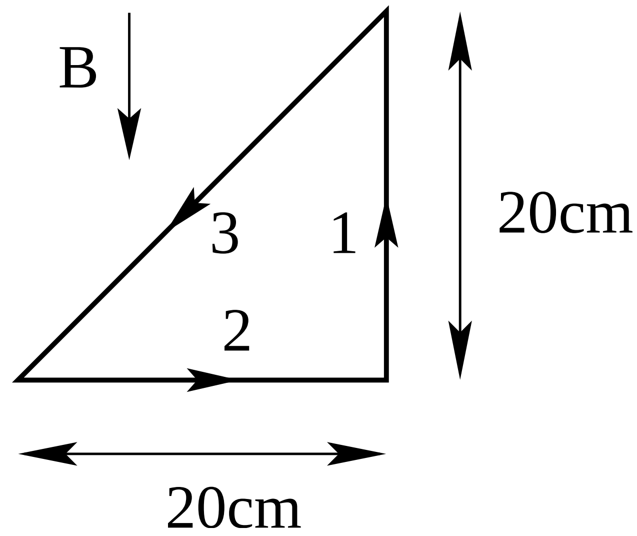
(b) $F = qvB\sin 30° = (5 \times 10^{-9}\mathrm{C})(3\mathrm{m/s})(4 \times 10^{-3}\mathrm{T})(0.5) = 3 \times 10^{-11}\mathrm{N}$.

Intermediate Exam II: Problem #4 (Spring ’06)

A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30$mT as shown. The current in the loop is $I = 0.4$A in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



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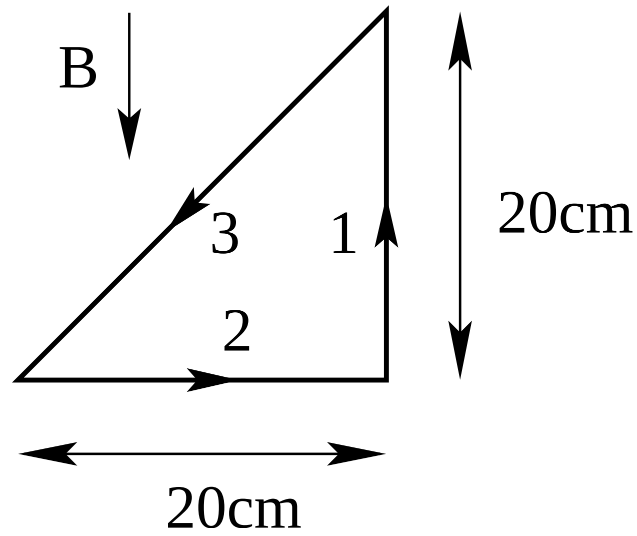

Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180°$).

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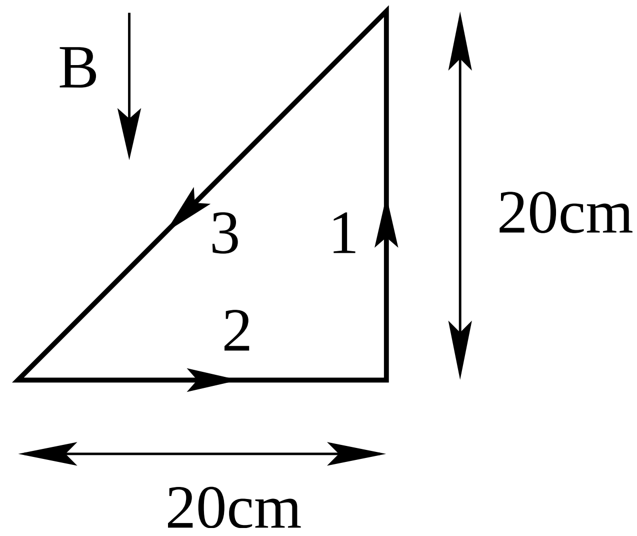


Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180°$).

(b) $F_2 = ILB = (0.4\text{A})(0.2\text{m})(30 \times 10^{-3}\text{T}) = 2.4 \times 10^{-3}\text{N}$.
Direction of $\vec{F}_2$: $\otimes$ (into plane).

Unit Exam III: Problem #1 (Spring ’12)

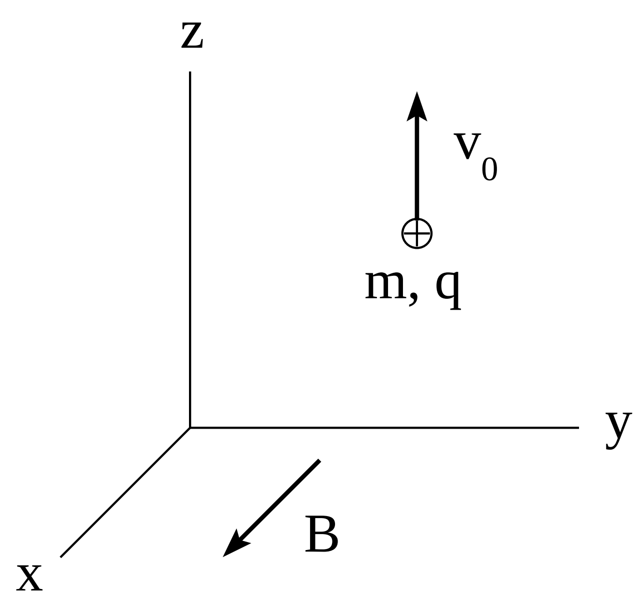
In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton ($m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C}$) is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius r of the circular path.

(c) Calculate the time T it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.



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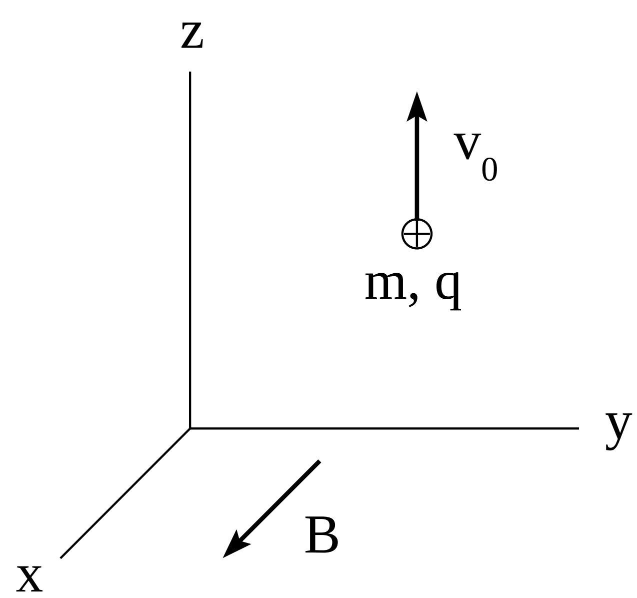

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

Unit Exam III: Problem #1 (Spring ’12)

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(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
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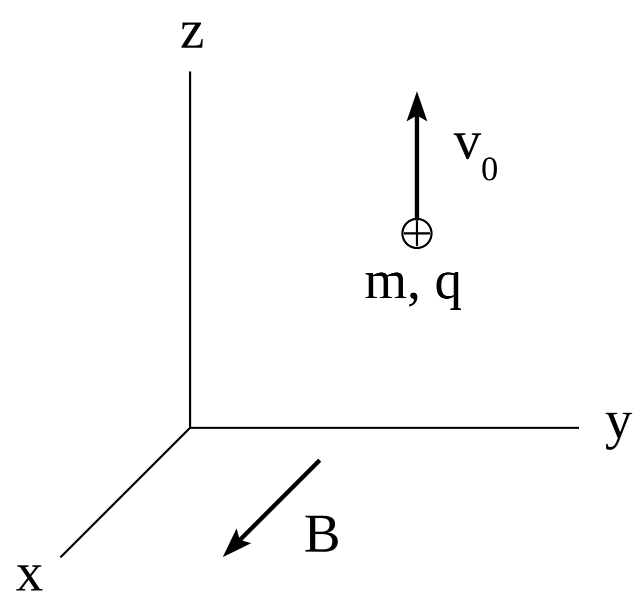
(d) Sketch the circular path of the proton in the graph.



Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton
$(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.
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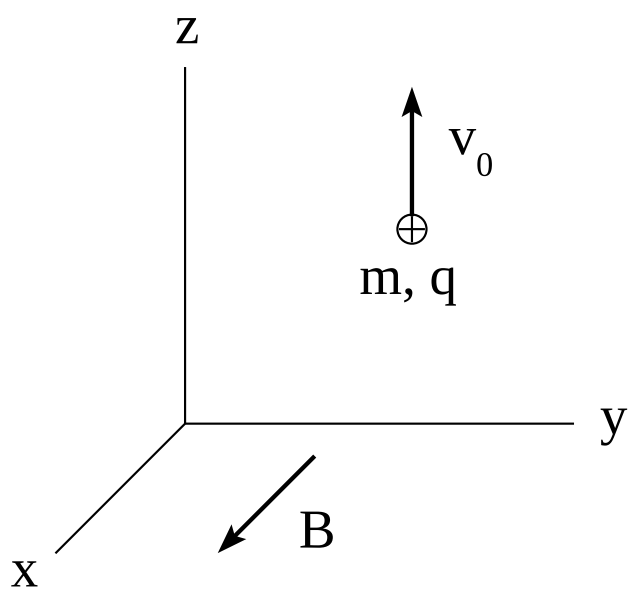


Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

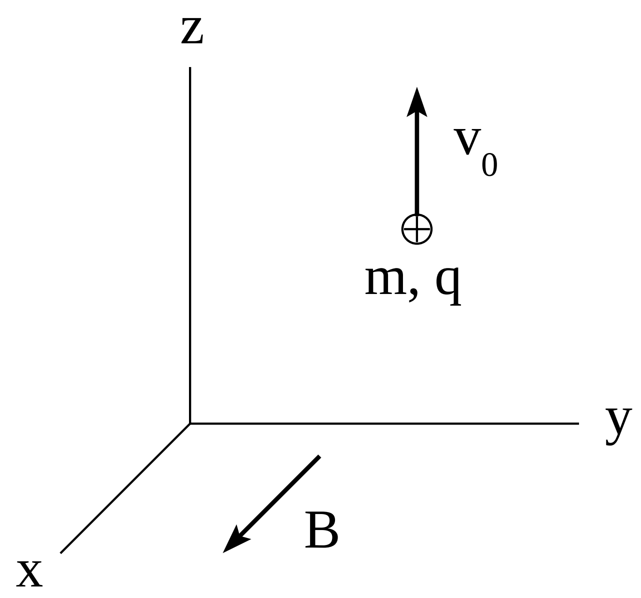
(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

(c) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi m}{qB} = 13.1\mu\mathrm{s}.$

Unit Exam III: Problem #1 (Spring ’12)

In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius r of the circular path.
(c) Calculate the time T it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.



Solution:

(a) $F = qv_0 B = 3.2 \times 10^{-18}\mathrm{N}.$

(b) $\dfrac{mv_0^2}{r} = qv_0 B \quad \Rightarrow \ r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

(c) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi m}{qB} = 13.1\mu\mathrm{s}.$

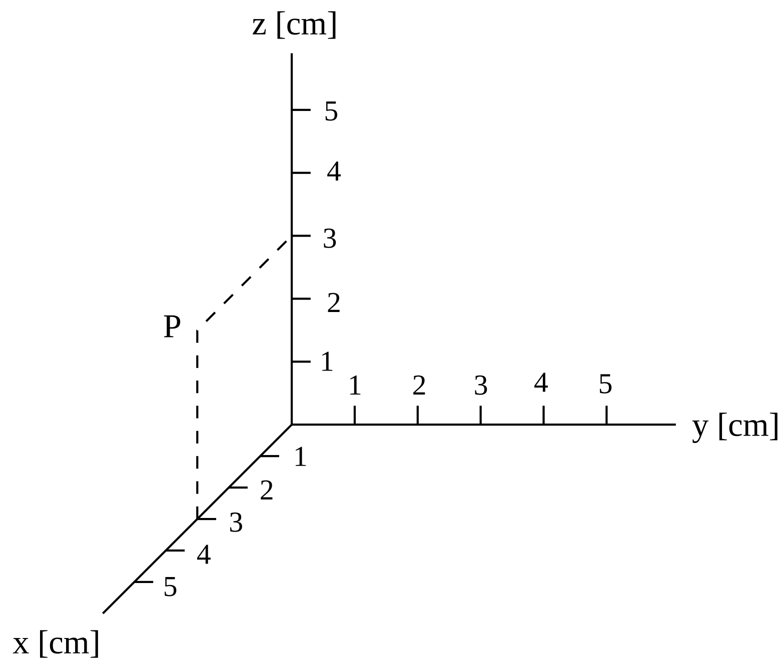
(d) Center of circle to the right of proton’s initial position (cw motion).

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\mathrm{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\mathrm{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



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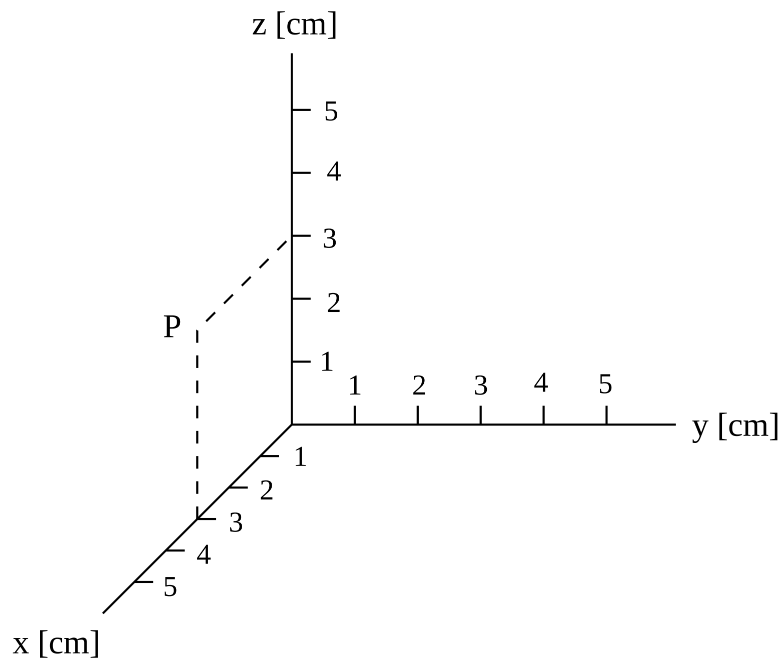

Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

Unit Exam III: Problem #1 (Spring ’13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

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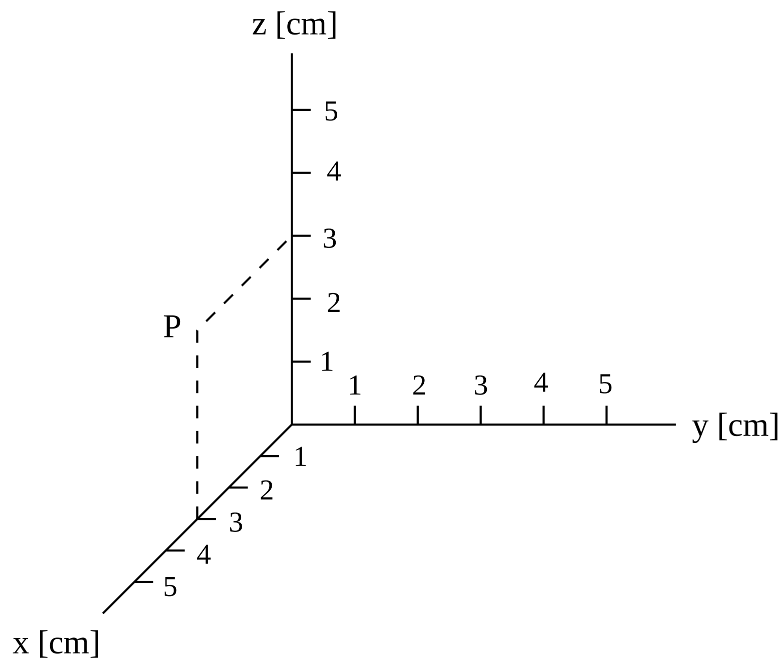

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(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow\ r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

Unit Exam III: Problem #1 (Spring '13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

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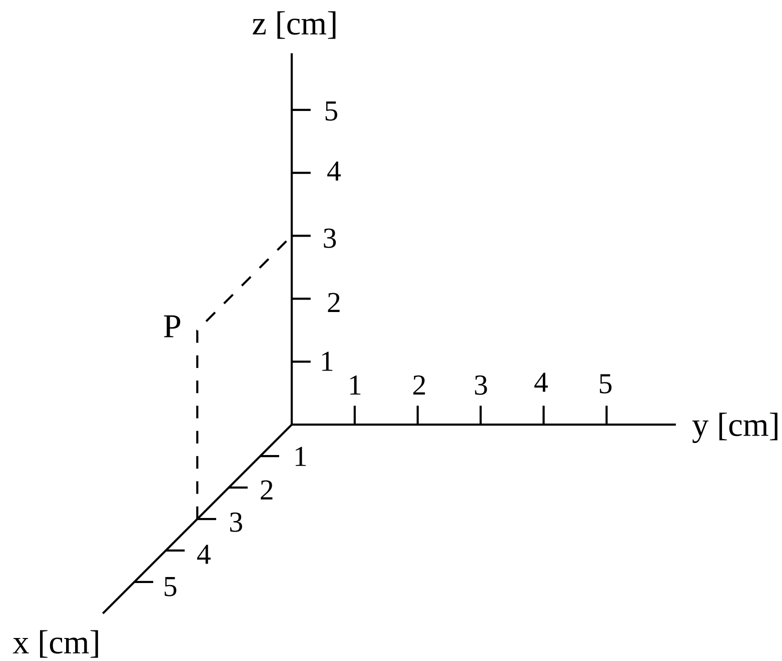

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$\Rightarrow \ \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow \ r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

(c) $C = 3.84\text{cm}\,\hat{\mathbf{i}} + 3.00\text{cm}\,\hat{\mathbf{k}}.$