## PROJECTILE MOTION

## Introduction:

After a gun fires a bullet, or for that matter, any device launches a projectile, the launched projectile follows a ballistic trajectory influenced by one force, gravity (neglecting air friction and any obstacles). Under such ideal conditions, the projectile should only experience a constant acceleration ' $\mathbf{g}$ ' in one direction, downward, and should maintain a constant velocity in all other directions perpendicular to $\mathbf{g}$. In other words, a ballistic projectile's vertical motion is similar to the freely falling body that you investigated in in the previous experiment. In this exercise, we will attempt to verify these assumptions by analyzing the kinematic motion of a projectile in flight.

This lab will be performed in two separate parts. During the first part, you will video the motion of a launched projectile (a plastic ball fired from a spring gun) and use special software to digitize the flight path. During the second part, you will complete the analysis of the motion by separating its components into vertical and horizontal motion. The goal of this analysis is to see if the horizontal component corresponds to a body at constant speed, while the vertical component is virtually the same as free fall. (What would convince you of this?)

After performing this lab you should:

1. Understand that a ballistic projectile's trajectory can be analyzed as two independent motions (vertical free fall and constant horizontal speed).
2. Get some understanding of a 'good fit' vs a 'bad fit' when fitting a function to data.
3. Identify the real-world parameters of the quadratic equation, eg., initial position, initial velocity, and acceleration.
4. Get familiar with the 'Logger Pro' video capture and analysis software.

## Part 1

## Setup:

When you arrive, the projectile launcher will be set up in front of a black background screen with a meter stick attached. This stick will be used to scale the distance measurements. The whole arrangement will be in the field of view of a video camera.

As in any experiment, be sure to carefully sketch the apparatus and label its parts, paying close attention to the instruments that yield measurements. Also, define the quantities you will measure.

Measure the distance between the camera and the background screen and the distance from the launcher to screen. Referring to the analysis graphs (see below) draw a coordinate system on your sketch and identify the X and Y -axes to be used by the video analysis software.

Each student will be responsible for firing a projectile at a different angle; for a group of three, use $35^{\circ}, 45^{\circ}$, and $55^{\circ}$.

Before you begin:
Make sure the spotlight is turned on and illuminating the screen.
Make a table for your data in your notebook with the information below. If your group is first, you can make the table later.

| Parameter | Value | Instrument | Zero | ssd | Random <br> error |
| :--- | :--- | :--- | :--- | :--- | :--- |
| D |  |  |  |  |  |
| D |  |  |  |  |  |
| L |  |  |  |  |  |
| $\theta$ |  |  |  |  |  |

where $\mathrm{d}=$ diameter of ball, $\mathrm{D}=$ distance from camera to screen, and $\mathrm{L}=$ distance from launcher to screen

Notes about the launcher: The device has three spring compression levels labeled on the launcher as, Short Range, Medium Range, and Long Range. When loading the projectile each level is easily distinguished by a distinctive click made by the latching mechanism. You will use the 'Medium Range' setting (2 clicks). The launch angle is read from a protractor and plumb line on the side of the device.

## Recording the launch and the flight of the projectile on Video:

When you are ready to record an event, it will be necessary to work as a team, one student controlling the launcher, while another starts the capture, and a third student catches the ball. (The catcher must wear protective goggles)

- Go to "Camera" program either in the start menu or pinned to the task bar
- Choose USB camera in the top right corner of the window
- Click the white button to start and stop recording of the video
- Replay your video to be sure it captured the projectile motion.
- To Trim the video:
- Select video in "Camera Roll" folder
- Click on the pencil in the bottom right corner of the video viewing window and select "Trim"
- Drag the bounds to just before the ball was fired and just after it leaves the frame of the video
- Click "Save Copy" in top right corner of video trim window -
- To save the file with a unique name:
- Go back to "Camera Roll" folder
- Rename the trimmed video file
- Click on the triangle to expand the "Save as Window" to show other shared locations and select the "Desktop/Remote Data Storage" folder on the server "Sal".
Repeat for different launch angles.


## Analysis:

- Open Logger Pro from your lab group's computer.
- Under the "Insert" menu, choose "Movie".
- Verify your video: Click on the "Desktop/Remote Data Storage" link and locate your video. Preview it to make sure the flight of the projectile is captured, i.e., step through each frame to find captured images of the projectile. Usually it's there, but starts late.
- Set the distance scale: This software requires a reference distance (in meters) close to the event being measured. For
this experiment, we will use the top and bottom of the meter stick taped to the background screen. To set the scale, toggle the icon in the lower right of the captured video to enable video analysis. Click on the "Set Scale" icon ( $4^{\text {th }}$ down on right). Drag the mouse between the top and bottom of the meter stick and enter " 1 " (meter). If the cursor is not tracking the image of the meter stick correctly, try slightly resizing the video.
- Digitize the projectile position: Click on the "Add point" icon ( $2{ }^{\text {nd }}$ down on right side) and use the mouse to position the cursor over the center of the ball. Click the mouse button to digitize the projectile's position in the frame (it will also automatically advance to the next frame). Continue for each frame in which the ball appears. If a dot is not appearing over the ball, click on the "Toggle Trails" icon ( $7^{\text {th }}$ down on the right hand side of the image). If the cursor is not tracking the projectile correctly, try slightly resizing the video.
- Now choose "Auto Arrange" under the "Page" menu to view data, plots and video.

Plots of the x - and y -positions versus time should be displayed automatically. To enable curve fitting within the 'Analyze' menu, click anywhere on the plotted graph. Once it is enabled, select "Curve Fit" and then "VideoAnalysis", and choose "Quadratic" fit, "Try Fit", and "OK". Also, click on the "time offset" box. Do the same for the $y$-position versus time data. Record your values from the fits of $\mathrm{V}_{\text {ox }}, \mathrm{V}_{\text {oy }}, \mathrm{a}_{\mathrm{x}}$, and $\mathrm{a}_{\mathrm{y}}$ in a table as shown below. Print out your data table and plots on separate printouts.

| $\mathrm{V}_{\text {ox }}(\mathrm{m} / \mathrm{s})$ | $\mathrm{V}_{\text {oy }}(\mathrm{m} / \mathrm{s})$ | $\mathrm{a}_{\mathrm{x}}=2 \mathrm{~A}_{\mathrm{x}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{a}_{\mathrm{y}}=2 \mathrm{~A}_{\mathrm{y}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Part 2

Add your data to a table on the board, one data table for each angle:

| Group | $\mathrm{A}_{\mathrm{x}}$ <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{A}_{\mathrm{y}}$ <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{V}_{\mathrm{ox}}$ <br> $(\mathrm{m} / \mathrm{s})$ | $\mathrm{V}_{\mathrm{oy}}$ <br> $(\mathrm{m} / \mathrm{s})$ | $\theta=\tan ^{-1}$ <br> $\left(\mathrm{~V}_{\mathrm{oy}} / \mathrm{V}_{\mathrm{ox}}\right)$ | $\theta_{\text {meas }}$ | $\mathrm{V}_{\mathrm{o}}=\mathrm{V}_{\mathrm{ox}}$ <br> $/ \cos \theta$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| $\ldots$ |  |  |  |  |  |  |  |

In the above you should calculate $V_{o}$ (last column) from $V_{o x}$ and your measured angle.

Record in your notebook the data table that corresponds to the angle you used. Find the mean and uncertainty of the values of $\mathrm{a}_{\mathrm{x}}$, and $\mathrm{a}_{\mathrm{y}}$, and record them in your notebook.

Draw linear comparison graphs for the experimental vs. accepted values of the acceleration in the $x$ - and $y$-directions. Use the $\pm$ from the curve fit box in Logger Pro for $A_{x}$ and $A_{y}$ multiplied by 2 to have $\Delta A_{x}$ and $\Delta A_{y}$ for the linear comparison graph.

## Error Analysis:

Random errors: Are the fitted curves good representations of the data? This is the same as asking "is the square root of the mean square error (mse), called the 'root mean square error' (rmse) of the fit approximately the same as the error of measuring the position?" If the answer is yes, then the fit is a good one, and the data is well represented by the fitted curve.

From the recorded size of the ball, estimate the precision with which you can place the blue dot over the ball's image in the digitizing process. Is it $1 / 10^{\text {th }}$ the size of the ball (about 2 mm )? Is it $1 / 2$ the size of the ball (about a centimeter)? You decide, write the number in your notebook, and
write a short sentence telling your reader why you picked this value. This is the error of your measurement.

Is the rmse of the fit about the same as this estimate? Is it much larger than the estimate of error? . . . much smaller? Is the fit a good one?

Bias: What is the effect of having the calibrating scale further from the camera than the actual trajectory of the projectile? If you can, Estimate the magnitude of this effect.

## Discussion:

1. Are the fits good? Explain.
2. Does the y-motion show constant acceleration? Explain.
3. Does the x-motion demonstrate constant velocity? Explain.
4. Are the class values of $\mathrm{A}_{\mathrm{yexp}}$ in agreement with the accepted value within the range of uncertainty? Explain.
5. Are the calculated and measured angles in agreement? Explain.
6. Are the values for $\mathrm{V}_{\mathrm{o}}$ consistent from group-to-group? Explain.
