[tex94] Classical ideal gas (grandcanonical ensemble)

Consider a classical ideal gas,

$$\mathcal{H}_N = \sum_{i=1}^{3N} \frac{p_i^2}{2m},$$

within a region of volume V in equilibrium in contact with heat reservoir at temperature T and a particle reservoir at chemical potential μ.
(a) Show that the grand partition function is $Z = \exp(zV/\lambda_T^3)$, where $z = \exp(\mu/k_BT)$ is the fugacity, and $\lambda_T = \sqrt{h^2/2\pi m k_B T}$ is the thermal wavelength.
(b) Derive from Z the grand potential $\Omega(T, V, \mu)$, the entropy $S(T, V, \mu)$. the pressure $p(T, V, \mu)$, and the average particle number $\langle N\rangle = \mathcal{N}(T, V, \mu)$.
(c) Derive from these expressions the familiar results for the internal energy $U = \frac{3}{2}\mathcal{N}k_BT$, and the ideal gas equation of state $pV = \mathcal{N}k_BT$.

Solution: