[tex92] Relativistic ideal gas (entropy and internal energy)

(a) Derive from the result for the canonical partition function Z_N of the relativistic classical ideal gas as calculated in [tex91] the Helmholtz free energy $A(T, V, N)$, the equation of state $p(T, V, N)$, the chemical potential $\mu(T, V, N)$, and the entropy $S(T, V, N)$.
(b) Use the recursion relation, $K_n'(u) = -K_{n-1}(u) - (n/u)K_n(u)$, for the modified Bessel functions to derive the following expression for the internal energy:

$$U(T, N) = Nk_BTu\left[\frac{K_1(u)}{K_2(u)} + \frac{3}{u} - 1\right], \quad u \equiv \beta mc^2, \quad \beta = \frac{1}{k_BT}.$$

Solution: