

[tex9] Work extracted from finite heat reservoir in infinite environment

A (finite) heat reservoir with heat capacity $C = \text{const}$ is initially at temperature T_H and the (infinite) environment at the lower temperature T_0 . Now the reservoir is connected to the environment by a heat engine, which absorbs an infinitesimal amount of heat dQ per cycle, converts part of it into work dW , and dumps the rest into the environment. During each cycle the temperature of the reservoir decreases infinitesimally: $dQ = -CdT$. The fraction of the excess internal energy $U_{ex} = C(T_H - T_0)$ that is converted into work is $\Delta W/U_{ex}$.

- (a) Determine the maximum amount of $\Delta W/U_{ex}$ that can be extracted from the reservoir before its temperature has dropped to that of the environment.
- (b) Plot this quantity versus the reduced temperature $(T_H - T_0)/T_0$ for $T_0 < T_H < 3T_0$.
- (c) Set $T_H/T_0 = 1 + \epsilon$ with $\epsilon \ll 1$ and find the dependence of $\Delta W/U_{ex}$ on ϵ to order $O(\epsilon^2)$.

Solution: