[tex84] Classical paramagnet (canonical ensemble)

Consider an array of N noninteracting localized magnetic dipole moments in the form of classical 3-component unit vectors $\mathbf{m}_i = (m_i^x, m_i^y, m_i^z) = (\sin\theta_i \cos\phi_i, \sin\theta_i \sin\phi_i, \cos\theta_i)$. In the presence of a magnetic field $\mathbf{H}$ pointing in z-direction, the Hamiltonian of this system represents the Zeeman energy:

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{m}_i \cdot \mathbf{H} = -H \sum_{i=1}^{N} m_i^z.$$

(a) Calculate the canonical partition function Z_N of this system.
(b) Calculate the Gibbs free energy $G(T, H, N)$, the magnetization $M(T, H, N)$ (Langevin function), the isothermal susceptibility $\chi_T(T, H, N)$, and the heat capacity $C_H(T, H, N)$.
(c) Show that the leading term in an expansion of χ_T at small H is H-independent and represents Curie's law $\chi_T \simeq N/3k_BT$.
Note that $\phi, \cos\theta$ are a pair of conjugate canonical coordinates.

Solution: