[tex78] Array of classical harmonic oscillators (canonical ensemble)

Consider an array of N 3-dimensional classical harmonic oscillators, representing a system of $3N$ uncoupled degrees of freedom:

$$H = \sum_{i=1}^{3N} \left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 q_i^2 \right).$$

(a) Calculate the canonical partition function Z_N for this model.

(b) Derive from Z_N the Helmholtz free energy $A(T, N)$, the entropy $S(T, N)$, the internal energy $U(T, N)$, and the heat capacity $C \equiv (\partial U/\partial T)_N$.

Solution: