[tex76] Classical ideal gas (canonical ensemble)

Consider a classical ideal gas of N atoms confined to a box of volume V in thermal equilibrium with a heat reservoir at temperature T. The Hamiltonian of the system reflects the kinetic energy of $3N$ noninteracting degrees of freedom:

$$H = \sum_{i=1}^{3N} \frac{p_i^2}{2m}.$$

(a) Show that the canonical partition function is $Z_N = V^N/(N!\lambda_T^{3N})$, where $\lambda_T = \sqrt{h^2/2\pi m k_B T}$ is the thermal wavelength.
(b) Derive from Z_N the Helmholtz free energy $A(T,V,N)$, the entropy $S(T,V,N)$, the pressure $p(T,V,N)$, the internal energy $U(T,N)$, and the chemical potential $\mu(T,V)$.
(c) Show that the pressure is equal to two thirds of the energy density and that the adiabates satisfy $p^3V^5 = \text{const}$.

Solution: