

[tex75] Quantum harmonic oscillators (microcanonical ensemble I)

Consider an array of  $N$  quantum harmonic oscillators:

$$H = \sum_{i=1}^N \left[ \hbar\omega \left( n_i + \frac{1}{2} \right) \right], \quad n_i = 0, 1, 2, \dots$$

- (a) Calculate the entropy  $S(U, N)$  of this system in the *microcanonical* ensemble via combinatorics as follows: Set  $U = U_0 + M\hbar\omega$ ,  $U_0 = \frac{1}{2}N\hbar\omega$ ,  $M = n_1 + \dots + n_N$ . Next determine the number  $N_{\Delta}(M, N)$  of configurations  $(n_1, \dots, n_N)$  for fixed values of  $M, N$ . Then relate  $S$  to  $N_{\Delta}$ .
- (b) Derive the internal energy  $U(T, N)$ , and the heat capacity  $C = (\partial U / \partial T)_N$  from  $S(U, N)$ .

**Solution:**