[tex72] Effect of escaping particles on temperature of 1D ideal gas

A classical ideal gas (particle mass m, particle density n, temperature T, energy density $u = \frac{1}{2}nk_BT$) is contained in the region $0 < x < L$. The particles can only move in $\pm x$-direction. We assume that the velocities satisfy a 1D Maxwell distribution at all times. All particles that hit the wall at $x = 0$ are reflected elastically. The wall at $x = L$ allows any particle that hits it to pass through with a probability $\epsilon_0 \ll 1$, independent of the particle's energy. Otherwise the particle is reflected elastically.
(a) Calculate the rate at which the system loses particles and energy. Express the rates in the form $dn/dt = f_n(n,T)$ and $du/dt = f_u(n,T)$.
(b) The slowly varying particle density $n(t)$ and energy density $u(t)$ cause a slowly varying temperature $T(t)$ of the remaining gas. Derive from the results of (a) a differential equation for the function $T(t)$ and solve it.

Solution: