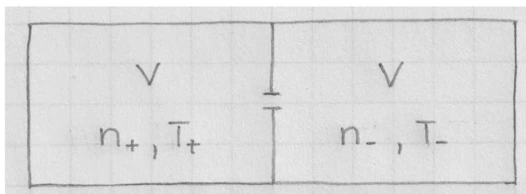
[tex64] Toward thermal equilibrium via particle transfer

A vessel with insulating walls is divided into two compartments by an internal wall that is also insulating, but has a small hole of area A. The two compartments contain dilute gases of slightly different densities, $n_\pm = n \pm \frac{1}{2}\Delta n$, at slightly different temperatures, $T_\pm = T \pm \frac{1}{2}\Delta T$. We set $\Delta n > 0$ and allow ΔT to be positive or negative.
(a) Show that the rates at which particles and energy are transferred through the hole are (in leading orders of Δn and ΔT):

$$\frac{dN}{dt} \doteq \frac{dN_-}{dt} - \frac{dN_+}{dt} = \frac{A}{\sqrt{2\pi m}}\left[\sqrt{k_BT}\Delta n + \frac{1}{2}\frac{n}{\sqrt{k_BT}}(k_B\Delta T)\right],$$

$$\frac{dE}{dt} \doteq \frac{dE_-}{dt} - \frac{dE_+}{dt} = \frac{A\sqrt{2}}{\sqrt{\pi m}}\left[(k_BT)^{3/2}\Delta n + \frac{3}{2}n\sqrt{k_BT}(k_B\Delta T)\right].$$

(b) If the compartment with the higher particle density is at the lower temperature, i.e. for $\Delta T < 0$, it is possible to create situations where either the particle flow or the energy flow is instantaneously zero. Find the values of $\Delta T/\Delta n$ in terms of n and T for which we have either $dN/dt = 0$ or $dE/dt = 0$ initially when the hole is opened.



Solution: