[tex63] Doppler broadening of atomic spectral lines

Consider a furnace containing a dilute gas at high temperature. Through a small window of the furnace, we observe a particular spectral line of the gas atoms by means of a spectrometer. The width of the observed spectral line is broadened due to the spread of velocities of the gas atoms. This effect is called *Doppler broadening*. The relativistic Doppler shift of the wavelength is $\lambda = \lambda_0\sqrt{(1+v/c)/(1-v/c)}$. For the case under consideration we can assume that $v/c \ll 1$. Show that the intensity profile is given by the expression

$$I(\lambda) \;\propto\; \exp\left(-\frac{mc^2(\lambda-\lambda_0)^2}{2\lambda_0^2 k_B T}\right),$$

where T is the temperature of the furnace, c is the speed of light, m is the mass of the gas atoms, and λ_0 is the wavelength of the radiation emitted by an atom at rest.

Solution: