## [tex47] Statistical concept of uncertainty

An experiment has $n$ possible outcomes that occur with probabilities $P_1, \ldots, P_n$. The uncertainty about the outcome of the experiment is defined as

$$\Sigma(P_1, \ldots, P_n) = -\sum_{i=1}^{n} P_i \ln P_i.$$

(a) Prove that the maximum uncertainty occurs if all $P_i$ are equal.
(b) The $n^2$ combined outcomes of two independent experiments have probabilities $P_{ij} = P_i^I P_j^{II}$. Show that the uncertainty about the combined outcome of the two independent experiments is equal to the sum of the uncertainties of the outcomes of each experiment: $\Sigma(\{P_{ij}\}) = \Sigma(\{P_i^I\}) + \Sigma(\{P_i^{II}\})$.

**Solution:**