[tex40] Equation of state and adiabate of an elastic band

Experimentally one finds the following response functions of a band of elastic material

$$\left(\frac{\partial J}{\partial L}\right)_T = \frac{aT}{L_0} \left[1 + 2\left(\frac{L_0}{L}\right)^3 \right], \quad \left(\frac{\partial J}{\partial T}\right)_L = \frac{aL}{L_0} \left[1 - \left(\frac{L_0}{L}\right)^3 \right],$$

where J is the tension, L is the length, T is the temperature, and a, L_0 are constants. The heat capacity at constant length is $C_L = \text{const.}$

- (a) Use this empirical information to reconstruct the equation of state J(T, L).
- (b) Calculate the response function $(\partial L/\partial T)_J$ and discuss its physical meaning.
- (c) If the band is stretched adiabatically from length L_0 at temperature T_0 to length $2L_0$, what is the final temperature?

Solution: