[tex4] Carnot cycle of an ideal paramagnet

Consider the four steps of a Carnot engine with the operating material in the form of an ideal paramagnet. The equation of state is Curie's law, $M = DH/T$, where H is the magnetic field, T the absolute temperature, and D a constant. The internal energy is a monotonically increasing function, $U(T)$, of temperature.
(a) Determine the heat transfer, ΔQ, the work performance, ΔW, and the change in internal energy, ΔU, for each of the four steps:

$1 \to 2$ *isothermal demagnetization*: $T = T_H = \text{const}$, $M_2 < M_1$.
$2 \to 3$ *adiabatic demagnetization*: $S = \text{const}$, $M_3 < M_2$.
$3 \to 4$ *isothermal magnetization*: $T = T_L = \text{const}$, $M_4 > M_3$.
$4 \to 1$ *adiabatic magnetization*: $S = \text{const}$, $M_1 > M_4$.

(b) Sketch the Carnot cycle in the (M, H)-plane and in the (U, S)-plane.
(c) Show that the efficiency is $\eta_C = 1 - T_L/T_H$.

Solution: