[tex37] Coexistence line of continuous phase transition

Consider 1 mol of a fluid with two phases 1 and 2 in coexistence. The coexistence line is $p(T)_{coex}$. Suppose that the volume V and the entropy S vary continuously at the transition ($\Delta S = 0$ and $\Delta V = 0$), but the response functions C_p (heat capacity at constant pressure), α_p (thermal expansivity), and κ_T (isothermal compressibility) are discontinuous. Now consider the differentials dS and dV for each phase and for paths in the (T, p)-plane. Then calculate $\Delta S \doteq dS^{(2)} - dS^{(1)}$ and $\Delta V \doteq dV^{(2)} - dV^{(1)}$ between points an infinitesimal distance across the coexistence line
(a) at constant p,
(b) at constant T.
In the limit where the distance between the two points shrinks to zero, the ratio $\Delta S/\Delta V$ stays finite and expresses (via Clausius-Clapeyron) the slope $(dp/dT)_{coex}$ of the coexistence line in terms of the discontinuities, $\Delta C_p, \Delta\alpha_p, \Delta\kappa_T$, in the response functions.
(c) Derive a relation between $\Delta C_p, \Delta\alpha_p, \Delta\kappa_T$ from the consistency condition of the results obtained in parts (a) and (b).

Solution: