[tex201] Ising lattice gas in $\mathcal{D} = 1$: entropy II

Here we explore the T-dependence of the entropy S for the lattice gas with the number of particles N_p kept constant. We start from the expressions $S(T, V, \mu)$ and $N_\mathrm{p}(T, V, \mu)$ established in [tex195]. Switching from the grandcanonical to the canonical ensemble means (effectively) expressing μ as a function of N_p and then substituting the result into the expression of $\bar{S}$.

To calculate and plot $\bar{S}/k_B$ as a function of k_BT/u for fixed N_p at $u \neq 0$ proceed as follows:
(i) Introduce the quantities $x \doteq e^{\beta u/2}$, $z \doteq e^{\beta\mu/2}$, and $n \doteq 2N_\mathrm{p} - 1$. The variables z, x, n are not independent.
(ii) Rewrite the relation $N_\mathrm{p}(T, V, \mu) = \frac{1}{2}$ as a relation between x, z, and n.
(iii) In this exercise we only consider the case $n = 0$ pertaining to half filling $N_\mathrm{p} = \frac{1}{2}N$, implying a functional relation $z(x)$ between the remaining two variables.
(iv) Write $\bar{S}/k_B$ as a function of x. It represents a universal curve entropy versus scaled temperature $k_BT/|u|$, which depends only on the strength of the (attractive or repulsive) interaction.

Solution: