[tex195] Ising lattice gas in $\mathcal{D} = 1$: entropy I

We consider a linear array of N cells of volume v_c. Each cell contains at most one particle. Particles in nearest-neighbor cells experience an attractive force, producing an interaction energy $-u$. The grand potential, established in [tex194], is

$$\Omega(T, V, \mu) = -N k_B T \left[w + \ln \left(\cosh w + \sqrt{\sinh^2 w + e^{-\beta u}} \right) \right], \tag{1}$$

where $w = \beta(u + \mu)/2$, $\beta = 1/k_B T$, and $N = V/v_c$.
(a) Derive a parametric expression for the entropy, $S/N k_B$ as a function of particle density N_p/N.
(b) Infer an explicit functional dependence of $S/N k_B$ on N_p/N in the ideal lattice gas limit $u \to 0$.

Solution: