

[tex187] Hosts and tags at level 1

Consider a system of N_A orbitals, where each orbital can be populated by one host, which, in turn, can accommodate an unlimited number of tags. The combinatorics of such a system is specified by two species of particles with the following interaction coefficients and capacity constants [tsc20]:

$$g_{hh} = 1, \quad g_{th} = -1, \quad g_{ht} = g_{tt} = 0; \quad A_h = N_A, \quad A_t = 0.$$

- (a) Derive a compact expression for the scaled configurational entropy $\bar{S} \doteq S/N_A$ as a function of the densities $\bar{N}_h \doteq N_h/N_A$ of hosts and $\bar{N}_t \doteq N_t/N_A$ of tags.
- (b) Produce of contour plot of the function $\bar{S}(\bar{N}_h, \bar{N}_t)$.
- (c) Determine the value of $\bar{N}_h^{(\max)}$ at fixed \bar{N}_t which maximizes \bar{S} . Include the function $\bar{N}_h^{(\max)}(\bar{N}_t)$ as a dashed line in the contour plot.
- (d) Construct from this information a function $\bar{S}_{\max}(\bar{N})$, which represents the maximum configurational entropy as a function of the total number $\bar{N} = \bar{N}_h + \bar{N}_t$ of particles. Produce a plot of $\bar{S}_{\max}(\bar{N})$. Mark and explain all landmarks on that curve.

Solution: