[tex186] Configurational entropy of statistically interacting particles

From the multiplicity of microstates,

$$W(\{N_m\}) = \prod_{m=1}^{M} \left(\begin{array}{c} d_m + N_m - 1 \\ N_m \end{array} \right), \quad d_m = A_m - \sum_{m'=1}^{M} g_{mm'}(N_{m'} - \delta_{mm'}),$$

where the A_m are capacity constants, the $g_{mm'}$ are statistical interaction coefficients, and the $\{N_m\}$ are the numbers of particles from M species present, use Stirling's approximation for factorials, $n! \simeq n \ln n - n$, to derive the following expression of the configurational entropy:

$$S(\{N_m\}) = k_{\mathrm{B}} \ln \big(W(\{N_m\})\big) = k_{\mathrm{B}} \sum_{m=1}^{M} \Big[\big(N_m + Y_m\big) \ln \big(N_m + Y_m\big) - N_m \ln N_m - Y_m \ln Y_m \Big],$$

$$Y_m = A_m - \sum_{m'=1}^{M} g_{mm'} N_{m'}.$$

Solution: