[tex183] Paramagnetic FD gas X: isothermal susceptibility at $\hat{H} = 0$.

For the isothermal susceptibility we established parametric representation [tex165],

$$\chi_{TV} = \frac{V}{k_B T \lambda_T^{\mathcal{D}}} \left[\frac{1}{f_{\mathcal{D}/2-1}(z_+)} + \frac{1}{f_{\mathcal{D}/2-1}(z_-)} \right]^{-1}.$$

(a) Show that can be simplified, in the limit $\hat{H} \to 0$, and upon scaling as spelled out in [tsc16], into the expression,

$$\hat{\chi}_T \doteq \left(\frac{\partial \bar{M}}{\partial \hat{H}} \right)_{\hat{T}} = \frac{1}{4\hat{T}} \frac{f_{\mathcal{D}/2-1}(z)}{f_{\mathcal{D}/2}(z)}, \quad \hat{T}^{-\mathcal{D}/2} = 2\Gamma(\mathcal{D}/2+1) f_{\mathcal{D}/2}(z).$$

(b) Produce graphical representations of $\hat{\chi}_T$ versus $\hat{T}$ for $\mathcal{D} = 1, 2, 3$ and comment on the salient features of the results in relation to the magnetization curves established in [tsc16].
(c) Establish the limit $\hat{T} \to 0$ of the above result for arbitrary $\mathcal{D}$.

Solution: