[tex182] Paramagnetic FD gas IX: exact magnetization curve for $\mathcal{D} = 2$.

(a) Extract, taking advantage of the simple structure of $f_1(z)$, from the paramagnetic representation,

$$\bar{M} = \frac{1}{2}\frac{f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) - f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)}{f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) + f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)},$$

$$\hat{T}^{-\mathcal{D}/2} = \Gamma(\mathcal{D}/2+1)\Big[f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) + f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)\Big],$$

of the function $\bar{M}(\hat{T}, \hat{H})$ derived in [tsc16] the following explicit result for $\mathcal{D} = 2$:

$$\bar{M}(\hat{T}, \hat{H}) = \frac{\hat{H}}{2} - \hat{T}\,\mathrm{Artanh}\left(\frac{\sinh(\hat{H}/2\hat{T})}{\sqrt{\sinh^2(\hat{H}/2\hat{T}) + e^{1/\hat{T}}}}\right).$$

(b) The low-temperature limit of this expression is subtle. The second term disappears if $\hat{H} < 1$, leaving the strictly linear first term, $\bar{M} = \hat{H}/2$. Show that for $\hat{H} > 1$, the second term becomes $(1 - \hat{H})/2$, producing a constant, saturated magnetization, $\bar{M} = 1/2$.

Solution: