[tex181] Paramagnetic FD gas VIII: magnetization curves at $T = 0$

(a) Extract, by means of asymptotic expansion, from the paramagnetic representation,

$$\bar{M} = \frac{1}{2}\frac{f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) - f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)}{f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) + f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)}, \tag{1}$$

$$\hat{T}^{-\mathcal{D}/2} = \Gamma(\mathcal{D}/2+1)\Big[f_{\mathcal{D}/2}\big(ze^{\hat{H}/2\hat{T}}\big) + f_{\mathcal{D}/2}\big(ze^{-\hat{H}/2\hat{T}}\big)\Big], \tag{2}$$

of the function $\bar{M}(\hat{T},\hat{H})$ derived in [tsc16] the following general inverse relation of the zero-temperature magnetization curve $\bar{M}(0,\hat{H})$:

$$\hat{H} = \left(\frac{1}{2}+\bar{M}\right)^{2/\mathcal{D}} - \left(\frac{1}{2}-\bar{M}\right)^{2/\mathcal{D}}$$

(b) Produce graphical representations of the results for $\mathcal{D} = 1, 2, 3, 5, 10$.
(c) Infer explicit analytic functions $\bar{M}(0,\hat{H})$ for $\mathcal{D} = 1, 2, 4, 8$.
(d) Calculate the initial slope of $\bar{M}(0,\hat{H})$ for arbitrary values of $\mathcal{D}$.

Solution: