[tex179] BE gas in $\mathcal{D}$ dimensions IX: entropy and internal energy

Starting from the expression of the grand potential in terms of the polylogarithmic BE functions as defined in [tsl36],

$$\Omega = -pV = -\frac{Vk_BT}{\lambda_T^{\mathcal{D}}} g_{\mathcal{D}/2+1}(z),$$

use the partial derivative, $S = -(\partial\Omega/\partial T)_{V,\mu}$, and then the relation, $U = TS - pV + \mu N$, to infer the following expression for entropy (left) and internal energy (right) at $T \geq T_c$ (top) and $T \leq T_c$ (bottom), respectively:

$$\frac{S}{Nk_B} = \begin{cases} \left(\dfrac{\mathcal{D}}{2}+1\right)\dfrac{g_{\mathcal{D}/2+1}(z)}{g_{\mathcal{D}/2}(z)} - \ln z, \\ \left(\dfrac{\mathcal{D}}{2}+1\right)\zeta(\mathcal{D}/2+1)\left(\dfrac{T}{T_v}\right)^{\mathcal{D}/2}, \end{cases} \qquad \frac{U}{Nk_BT_v} = \begin{cases} \dfrac{\mathcal{D}}{2}\dfrac{g_{\mathcal{D}/2+1}(z)}{g_{\mathcal{D}/2}(z)}\dfrac{T}{T_v}, \\ \dfrac{\mathcal{D}}{2}\zeta(\mathcal{D}/2+1)\left(\dfrac{T}{T_v}\right)^{\mathcal{D}/2+1}. \end{cases}$$

Solution: