[tex178] Entropy and internal energy from state occupancies

Start from the expressions for the grand potential Ω as a sum over 1-particle states given in [tsc13].
(a) Derive via partial derivative the entropy expressions,

$$S = \begin{cases} -k_B \sum_{k=1}^{\infty} \big[\langle n_k\rangle \ln\langle n_k\rangle + (1-\langle n_k\rangle)\ln(1-\langle n_k\rangle)\big] & : \text{(FD)} \\ -k_B \sum_{k=1}^{\infty} \big[\langle n_k\rangle \ln\langle n_k\rangle - (1+\langle n_k\rangle)\ln(1+\langle n_k\rangle)\big] & : \text{(BE)} \\ -k_B \sum_{k=1}^{\infty} \big[\langle n_k\rangle \ln\langle n_k\rangle - \langle n_k\rangle\big] & : \text{(MB)} \end{cases}$$

(b) Use the relation $U = \Omega + TS + \mu\mathcal{N}$ to derive the following expressions for the internal energy:

$$U = \sum_{k=1}^{\infty} \epsilon_k \langle n_k\rangle, \quad \langle n_k\rangle = \begin{cases} \dfrac{1}{z^{-1}e^{\beta\epsilon_k}+1} & : \text{ FD,} \\ \dfrac{1}{z^{-1}e^{\beta\epsilon_k}-1} & : \text{ BE,} \\ ze^{-\beta\epsilon_k} & : \text{ MB.} \end{cases}$$

Solution: