[tex177] Ideal gas atoms escaping from a container III

A large vessel of volume V initially contains N_0 atoms of a classical ideal gas in thermal equilibrium at initial temperature T_0. The rate at which the number N of atoms decreases due to particles escaping into the vacuum through a tiny hole of area A in a wall and the rate at which the escaping particles export energy is worked out in [tex62]:

$$\frac{dN}{dt} = -An\sqrt{\frac{1}{2\pi m}}(k_BT)^{1/2}, \quad \frac{dE}{dt} = An\sqrt{\frac{2}{\pi m}}(k_BT)^{3/2}.$$

If the container walls are insulating, then the temperature of the gas in the vessel will drop as particles escape from it, the reason being that faster particles are more likely to escape.
(a) Find the temperature $T(t)$ during the process of slow particle leakage. Use the above two rates and the principle of energy conservation to set up a differential equation for T.
(b) Find the density $n(t)$ of particles remaining in the vessel if the initial value is $n_0 = N_0/V$.
(c) Find the energy $E(t)$ exported in the process and show that its total $E(\infty)$ is qual to the initial internal energy.

Solution: