[tex169] Ultrarelativistic classical ideal gas (grandcanonical ensemble)

Consider an ultrarelativistic classical ideal gas,

$$\mathcal{H}_N = \sum_{l=1}^{N} |\mathbf{p}_l| c,$$

within a region of volume V in equilibrium in contact with heat reservoir at temperature T and a particle reservoir at chemical potential μ.
(a) Calculate the grand partition function $Z(T, V, z)$, where $z = \exp(\mu/k_B T)$ is the fugacity.
(b) Derive from Z the grand potential $\Omega(T, V, \mu)$, the entropy $S(T, V, \mu)$. the pressure $p(T, V, \mu)$, and the average particle number $\langle N \rangle = \mathcal{N}(T, V, \mu)$.
(c) Derive from these expressions the familiar results for the internal energy U, and the ideal gas equation of state, $pV = \mathcal{N} k_B T$.
(d) Perform the Legendre transform, $A(T, V, \mathcal{N}) = \Omega(T, V, \mu) + \mu N$, to recover the Helmholtz potential derived in [tex77] for the canonical ensemble.

Solution: