[tex163] Paramagnetic FD gas III: heat capacity C_{VM}

There are two alternative and equivalent definitions of this heat capacity:

$$C_{VM} \doteq \left(\frac{\partial U}{\partial T}\right)_{VMN}, \quad C_{VM} \doteq T\left(\frac{\partial S}{\partial T}\right)_{VMN}. \tag{1}$$

Show that both definitions lead to the same parametric expression,

$$C_{VM} = \frac{\mathcal{D}}{2}\sum_{\sigma=\pm} N_\sigma k_B \left[\left(\frac{\mathcal{D}}{2}+1\right)\frac{f_{\mathcal{D}/2+1}(z_\sigma)}{f_{\mathcal{D}/2}(z_\sigma)} - \frac{\mathcal{D}}{2}\frac{f_{\mathcal{D}/2}(z_\sigma)}{f_{\mathcal{D}/2-1}(z_\sigma)}\right], \tag{2}$$

irrespective of whether we start from [tex162] using the first definition or from [tex161] using the second definition.

Solution: