[tex157] Quantum paramagnet (three-level system)

Consider an array of N noninteracting localized magnetic dipole moments $\mathbf{m}_i$. Each magnetic moment may be produced, for example, by two unpaired electron spins localized in a paramagnetic insulator. In the presence of a magnetic field $\mathbf{H}$ pointing in z-direction, the Hamiltonian of this system represents the Zeeman energy:

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{m}_i \cdot \mathbf{H} = -H \sum_{i=1}^{N} m_i^z, \quad m_i^z = 1, 0, -1.$$

(a) Calculate the canonical partition function Z_N of this system.
(b) Derive the Gibbs free energy $G(T, H, N)$, the magnetization $M(T, H, N)$, and the entropy $S(T, H, N)$.
(c) Show that the internal energy U vanishes identically.
(d) Calculate the isothermal susceptibility χ_T at $H = 0$ as a function of T.
(e) Sketch the shape of adiabates in the (H, M)-plane and in the (H, T)-plane.

Solution: