[tex151] Adiabatic atmosphere

Consider a column of air [molar mass $M = 29\text{g}$] treated as a classical ideal gas [$pV = nRT$, $C_p/C_V = \gamma = 1.41$] in a uniform gravitational field $g = 9.81\text{m/s}^2$. The column is assumed to be in mechanical equilibrium but not (yet) in thermal equilibrium. The mechanical equilibrium is established by gravitational pressure and governed by the adiabatic relation pV^γ =const.
(a) Calculate the dependence on height z of the pressure p, the mass density ρ, and the temperature T, assuming that $p = p_0$ and $T = T_0$ at $z = 0$.
(b) Find the height z_m, expressed as a function of T_0, at which T, p, and ρ all reach zero. What is that height (in meters) if T_0 is room temperature?
Hints: (i) Infer from pV^γ =const the differential relation $dT/T = [(\gamma - 1)/\gamma]dp/p$. (ii) Use the relation $dp(z) = -\rho(z)d\mathcal{U}(z)$ from [tex150] linking pressure, mass density, and gravitational potential to infer differential equations for $T(z)$ and $p(z)$.

Solution: