[tex130] BE gas in $\mathcal{D}$ dimensions VIII: speed of sound

(a) Start from the relation $c = (\rho\kappa_S)^{-1/2}$ for the speed of sound as established in [tex18], where $\rho = m/v$ is the mass density and κ_S the adiabatic compressibility. Use general thermodynamic relations between response functions to derive the following expression for c in terms of dimensionless quantities:

$$\frac{mc^2}{k_BT} = \frac{(v/v_T)}{(p_T\kappa_T)}\left[1 + \frac{(T/T_p)^2(v/v_T)(T_p\alpha_p)^2}{(p_T\kappa_T)(C_V/\mathcal{N}k_B)}\right],$$

where v_T, p_T, T_p are defined in [tln71]. (b) Use the expressions derived in [tex129] for α_p, in [tex128] for κ_T, and in [tex97] for C_V to derive the result

$$\frac{mc^2}{k_BT} = \gamma\,\frac{g_{\mathcal{D}/2+1}(z)}{g_{\mathcal{D}/2}(z)}, \quad \gamma = 1 + \frac{2}{\mathcal{D}}.$$

(c) Relate the T-dependence of mc^2 to that of the isochore for $v = \text{const}$ and to that of the isobar for $p = \text{const}$.

Solution: