[tex13] Room heater: Electric radiator versus heat pump

A room is to be kept at temperature $T_H = 294\text{K}, (21°\text{C})$. The outdoor temperature is T_L. Heat, which leaks through the windows and doors at the rate $\bar{Q}_{leak} = \gamma \Delta T$ (Fourier's law), must be replaced by a room heater at the same rate. The electric radiator converts electric power $\bar{W}_{el}$ into heat $\bar{Q}_{el}$ with 100% efficiency. The electric heat pump uses the amount $\bar{W}_{sup}$ of electric power to drive a Carnot cycle in the reverse, which extracts heat $\bar{Q}_L$ at temperature T_L from the exterior and converts it (reversibly) into heat $\bar{Q}_H = \bar{Q}_L + \bar{W}_{hp}$ at temperature T_H. In the relation $\bar{W}_{hp} = (1 - \lambda)\bar{W}_{sup}$, λ represents the energy loss in the gears of the heat pump. Quantities with overbars denote energy transfers per time unit.
(a) Find $\bar{W}_{el}$ as a function of γ, T_H, T_L, and $\bar{W}_{sup}$ as a function of $\gamma, \lambda, T_H, T_L$.
(b) Plot $\bar{W}_{el}/\gamma$ and $\bar{W}_{sup}/\gamma$ versus $t_L \equiv T_L - 273\text{K}$ (measured in °C) for fixed $T_H = 294\text{K}$ and $\lambda = 0.8$ (20% efficiency).
(c) Determine the range of T_L for which the heat pump is more economical than the radiator.

Solution: