[tex128] BE gas in $\mathcal{D}$ dimensions VI: isothermal compressibility

(a) Show that the isothermal compressibility, $\kappa_T = -(1/V)(\partial V/\partial p)_{T\mathcal{N}}$, of the ideal BE gas in $\mathcal{D}$ dimensions at $T > T_c$ is

$$p_T \kappa_T = \frac{g'_{\mathcal{D}/2}(z)}{g_{\mathcal{D}/2}(z) g'_{\mathcal{D}/2+1}(z)}, \quad \frac{v}{v_T} = \frac{1}{g_{\mathcal{D}/2}(z)},$$

where $v \doteq V/\mathcal{N}$, $v_T \doteq (\Lambda/k_B T)^{\mathcal{D}/2}$, $p_T \doteq k_B T/v_T$, $\Lambda \doteq h^2/2\pi m$, and $g_n(z)$ are BE functions. Use $z g'_n(z) = g_{n-1}(z)$ for $n \geq 1$ to simplify the results in $\mathcal{D} \geq 2$. (b) Sketch $p_T \kappa_T$ versus v/v_T for $v \geq 0$ in $\mathcal{D} = 1$ and for $v \geq v_c$ in $\mathcal{D} = 3$, where $v_c/v_T = [\zeta(\mathcal{D}/2)]^{-1}$ marks the onset of BEC. (c) Determine the nature of the singularity of κ_T as $v/v_T \to 0$ in $\mathcal{D} = 1, 2$. Determine the critical compressibility $p_T \kappa_T$ at $v = v_c$ in $\mathcal{D} = 3, 5$.

Solution: