[tex122] Unstable white dwarf

Consider a burnt-out white dwarf star of the same composition as described in [tex121] but with N so large that most of the electrons are ultrarelativistic, $\epsilon \simeq cp = \hbar k c$, in the fully degenerate state.

(a) Under similar assumptions as in [tex121] show that the expression of the total energy now reads

$$E = E_{kin} + E_{pot} = \frac{\hbar c}{3\pi} \left(\frac{9\pi}{4}\right)^{4/3} \frac{N^{4/3}}{R} - \frac{12}{5} m_n^2 G \frac{N^2}{R},$$

where c is the speed of light.

(b) Find the critical mass in units of the solar mass, $m_c/m_\odot$, beyond which this star is unstable and thus prone to a gravitational collapse into a neutron star or a black hole.

Solution: