[tex120] FD gas in $\mathcal{D}$ dimensions: isotherm and adiabate

(a) Show that the isotherm of an ideal Fermi-Dirac gas in $\mathcal{D}$ dimensions is described by the parametric relation

$$\frac{p}{p_T} = f_{\mathcal{D}/2+1}(z), \qquad \frac{v}{v_T} = [f_{\mathcal{D}/2}(z)]^{-1},$$

where $v_T = \lambda_T^{\mathcal{D}}$ and $p_T = gk_BT/\lambda_T^{\mathcal{D}}$ are convenient reference values and $v = gV/\mathcal{N}$ is the reduced volume. (b) Show that the adiabate is described by the relation $pv^{(\mathcal{D}+2)/\mathcal{D}} = \text{const}$ for all values of v/v_0. (c) Show that the relation for the isotherm approaches Boyle's law, $pv = \text{const}$, for $v \gg v_T$ and that it approaches the adiabate, $pv^{(\mathcal{D}+2)/\mathcal{D}} = \text{const}$, for $v \ll v_T$.

Solution: