[tex117] FD gas in $\mathcal{D}$ dimensions: chemical potential I

(a) Start from the fundamental thermodynamic relation $\mathcal{N} = (gV/\lambda_T^{\mathcal{D}})f_{\mathcal{D}/2}(z)$ for the ideal Fermi-Dirac gas in $\mathcal{D}$ dimensions and use the reference temperature $k_BT_v = \Lambda/v^{2/\mathcal{D}}$, $v \doteq gV/\mathcal{N}$, $\Lambda \doteq h^2/2\pi m$ to derive the following parametric expression for the dependence on temperature T of the chemical potential μ:

$$\frac{\mu}{k_BT_v} = \frac{T}{T_v}\ln z, \qquad \frac{T}{T_v} = [f_{\mathcal{D}/2}(z)]^{-2/\mathcal{D}}.$$

(b) Derive the following expression for the Fermi energy ϵ_F and the Fermi temperature T_F:

$$\lim_{T\to 0}\frac{\mu(T)}{k_BT_v} = \frac{\epsilon_F}{k_BT_v} = \frac{T_F}{T_v} = [\Gamma(\mathcal{D}/2+1)]^{2/\mathcal{D}}.$$

(c) Show that this result includes the familiar result, $\epsilon_F = (h^2/2m)(3\mathcal{N}/4\pi gV)^{2/3}$ for $\mathcal{D} = 3$.

Solution: