[tex115] BE gas in $\mathcal{D}$ dimensions III: isotherm and isobar

(a) From the fundamental thermodynamic relations for the Bose-Einstein gas in $\mathcal{D} > 2$ dimensions (see [tln67]), derive the following expressions for the isotherm at $v > v_c$ and the isobar at $T \leq T_c$:

$$\frac{p}{p_T} = g_{\mathcal{D}/2+1}(z), \qquad \frac{v}{v_T} = [g_{\mathcal{D}/2}(z)]^{-1};$$

$$\frac{v}{v_p} = \frac{\left[g_{\mathcal{D}/2+1}(z)\right]^{\mathcal{D}/(\mathcal{D}+2)}}{g_{\mathcal{D}/2}(z)}, \qquad \frac{T}{T_p} = \left[g_{\mathcal{D}/2+1}(z)\right]^{-2/(\mathcal{D}+2)}.$$

where $v_T = (\Lambda/k_BT)^{\mathcal{D}/2}$, $p_T = \Lambda(k_BT/\Lambda)^{\mathcal{D}/2+1}$, $k_BT_p = \Lambda(p/\Lambda)^{2/(\mathcal{D}+2)}$, $v_p = (\Lambda/p)^{\mathcal{D}/(\mathcal{D}+2)}$ with $\Lambda \doteq h^2/2\pi m$ are convenient reference values for temperature and pressure and reduced volume. (b) Calculate the leading correction to the Maxwell-Boltzmann result for the isotherm at low density and for the isobar at high temperature.

Solution: